

Unit 9: Solving Systems of Linear Equations Algebraically

9.1 Solving Systems of Linear Equations by Substitution

Substitution Method:

Ex. Solve

$$4x + 5y = 26$$

$$3x = y - 9$$

$$(1) (3x+9) = y$$

check

$$4(-1) + 5(6) = 26$$

$$(2) 4x + 5(3x+9) = 26 \quad -4 + 30 = 26$$

$$4x + 15x + 45 = 26$$

$$19x + 45 = 26$$

$$-45 \quad -45$$

$$19x = -19$$

$$x = -1$$

$$(3) 3(-1) = y - 9$$

Ex. Solve

$$2x + y = 13$$

$$x - 0.4y = -16$$

$$10x - 4y = -160$$

$$(2) 10x - 4(-2x+13) = -160$$

$$10x + 8x - 52 = -160$$

$$18x - 52 = -160$$

$$+52 \quad +52$$

$$18x = -108$$

$$x = -6$$

$$(3) 2(-6) + y = 13$$

$$-12 + y = 13$$

$$y = 25$$

$$(-6, 25)$$

$$(4) \text{ch } 2(-6) + 25 = 13$$

$$-12 + 25 = 13$$

$$13 = 13 \checkmark$$

$$(-6) - 0.4(25) = -16$$

$$-6 - 10 = -16$$

$$-16 = -16 \checkmark$$

Steps

- Isolate a single variable in one of the two equations (If possible choose a variable with a coefficient of 1)

Substitute the expression into the other equation and solve one of the variables

Substitute the first variable solution into either original equation and solve for the remaining variable

Verify your solution by substituting into both original equations

LCM: 15

Ex. Solve

$$\frac{2}{3}x + \frac{3y}{5} = -2$$

$$\frac{3}{6}x - \frac{6y}{2} = -4$$

$$(2) 10(3y-42) + 9y = -30$$

$$30y - 420 + 9y = -30$$

$$39y - 420 = -30$$

$$39y = 390$$

$$y = 10$$

$$(3) x - 3(10) = -42$$

$$x - 30 = -42$$

$$x = -12$$

$$(-12, 10)$$

$$(1) 10x + 9y = -30$$

$$x - 3y = -42$$

$$x = (3y - 42)$$

$$(4) \frac{2}{3}(-12) + \frac{3(10)}{5} = -2$$

$$-8 + 6 = -2$$

$$-2 = -2 \checkmark$$

$$\text{ch } \frac{-12}{6} - \frac{10}{2} = -7$$

$$-2 - 5 = -7$$

$$-7 = -7 \checkmark$$

Ex. At a dance recital, there were 220 people. Tickets cost \$9 for an adult and \$6 for a child. The dance collected \$1614 in ticket sales. How many adults and how many children attended the recital?

Let $a = \#$ of adults
Let $c = \#$ of children

①

$$\begin{array}{r} 220 = a + c \\ -c \quad -c \\ \hline \end{array}$$

$$6c + 9a = 1614$$

$$(220 - c) = a$$

②

$$6c + 9(220 - c) = 1614$$

$$6c + 1980 - 9c = 1614$$

$$\begin{array}{r} -3c = -366 \\ -3 \quad -3 \end{array}$$

$$c = 122$$

③

$$220 = a + 122$$

$$98 = a$$

④ ^{ch}

$$220 = 98 + 122$$

$$220 = 220 \checkmark$$

ch.

$$6(122) + 9(98) = 1614$$

$$732 + 882 = 1614$$

$$1614 = 1614 \checkmark$$

98 adults
122 children

4.2 Solving Systems of Linear Equations by Elimination**Elimination Method:****Ex. Solve**

$6x = 24 - 5y$

$3y + 4x = 12$

$$\begin{array}{r} 6x + 5y = 24 \quad \times 2 \rightarrow 12x + 10y = 48 \\ 4x + 3y = 12 \quad \times 3 \rightarrow 12x + 9y = 36 \\ \hline 0x + y = 12 \end{array}$$

$y = 12$

$6x = 24 - 5(12)$

$6x = 24 - 60$

$6x = -36$

$x = -6$

$(-6, 12)$

 ch

$6(-6) = 24 - 5(12)$

$-36 = 24 - 60$

$-36 = -36 \checkmark$

$\text{ch } 3(12) + 4(-6) = 12$

$36 + -24 = 12$

$12 = 12 \checkmark$

- Rearrange the equations so that like variables appear in the same position in both equations. The most common form is $ax + by = c$.

- Determine which variable to eliminate. If necessary, multiply one or both equations by a constant to eliminate the variable by addition or subtraction.

- Solve for the remaining variable.

- Solve for the second variable by substituting the value for the first variable into one of the original equations.

- Verify your answer by substituting each value into both original equations.

Ex. Solve

$3a - 5b = -9$

$4a + 5b = 23$

 1

$7a = 14$

$a = 2$

$4(2) + 5b = 23$

$8 + 5b = 23$

$5b = 15$

$b = 3$

$(2, 3)$

 ch

$3(2) - 5(3) = -9$

$6 - 15 = -9$

$-9 = -9 \checkmark$

$4(2) + 5(3) = 23$

$8 + 15 = 23$

$23 = 23 \checkmark$

Ex. Solve

$y = 4x + 11$

$x - y = 1$

 3

$-4 - y = 1$

$-y = 5$

$y = -5$

 2

$-3x = 12$

$x = -4$

$(-4, -5)$

Ex. Connor downloaded two orders of games and songs. The first order consisted of five games and four songs for \$26. The second order consisted of three games and two songs for \$15. All games cost the same amount and all songs cost the same amount. Write a system of linear equations. Then, determine the cost of one song and the cost of one game.

Let g = cost of game Let s = cost of song

$$\begin{aligned} 5g + 4s &= 26 \rightarrow 5g + 4s = 26 \\ 3g + 2s &= 15 \xrightarrow{\times 2} -6g - 4s = -30 \end{aligned}$$

$$3(4) + 2s = 15$$

$$12 + 2s = 15$$

$$2s = 3$$

$$s = \frac{3}{2}$$

$$s = \$1.50$$

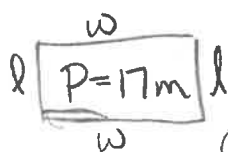
$$-1g = -4$$

$$g = \$4$$

$$1 \text{ game} = \$4$$

$$1 \text{ song} = \$1.50$$

Ex. The perimeter of a rectangular garden is 17 m. Triple the length is 2.46 m longer than five times the width. Sketch and label a diagram. Create a system of linear equations to determine the dimensions of the rectangle. Solve the system using elimination.



Let l = length of garden

w = width

$$\begin{aligned} \textcircled{1} \quad 2l + 2w &= 17 \rightarrow (2l + 2w = 17) \times 3 \\ \textcircled{2} \quad 3l &= 5w + 2.46 \rightarrow (3l - 5w = 2.46) \times -2 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad 6l + 6w &= 51 \\ -6l + 10w &= -4.92 \end{aligned}$$

$$16w = 46.08$$

$$w = 2.88 \text{ m}$$

$$\textcircled{4} \quad 2l + 2(2.88) = 17$$

$$2l + 5.76 = 17$$

$$2l = 11.24$$

$$l = 5.62 \text{ m}$$