



Unit 3: Factoring

3.4B.1 Common Factors

Greatest Common Factor (GCF):

Ex. Determine the GCF of each pair of terms:

a) $16x^2y$ and $24x^2y^3$ b) $5m^2n$ and $15mn^2$ c) $48ab^3c$ and $36a^2b^2c^2$

Recall distribution: $2x(3x^2 - 4) =$

Factoring is the reverse process:

To factor means to write as a product.

Ex. Write each polynomial in factored form.

a) $16r^2 - 20r$ c) $27r^2s^2 - 18r^3s^2 - 36rs^3$

b) $4x^3y^2 - 14x^2$ d) $12n^5p^4 + 8n^4p^3 - 4n^3p^2$

Recall double distribution: $(x - 3)(2x + y) =$ Step 1

Step 2

We can reverse step 1 by factoring a binomial factor:

Ex. Factor each polynomial.

a) $3x(x - 4) + 5(x - 4)$

b) $4(x + 5) - 3x(x + 5)$

We can reverse step 2 by group factoring:

Ex. Factor each polynomial.

a) $y^2 + 8xy + 2y + 16x$

b) $a^2 - a + 3ab - 3b$

Ex. Paula has 18 toonies, 30 loonies, and 48 quarters. She wants to group her money so that each group has the same number of each coin and there are no coins leftover.

a) What is the maximum number of groups she can make?

b) How many of each coin will be in each group?

c) How much money will each group be worth?

Practice: pg.

1

2

3

4

34B.3 Decomposition Factoring**Recall:**

$$(2x - 3)(x + 2) =$$

 $=$ $=$ **Decomposition Factoring**Factor. $3x^2 + 8x + 4$ **Step 1: Check for a GCF****Step 2: Find two integers with**

- A product of $3 \times 4 = 12$
- A sum of 8

Step 3: Split (decompose) the middle term into two parts using the integers from step 1**Step 4: Factor by grouping****Check?****Ex. Factor each trinomial, if possible:**

a) $2x^2 + 7x - 4$

c) $24x^2 - 30x - 9$

b) $3x^2 + 2x + 4$

d) $6x^2 - 5xy + y^2$

e) $-3a^3b - 51a^2b - 30ab$

f) $2y^2 + 7xy + 3x^2$

Ex. Identify all values of q that allows each trinomial to be factored:

a) $2x^2 + qx + 5$

b) $3x^2 + qxy - 4y^2$

Ex. Identify two values of q that allows each trinomial to be factored:

a) $2a^2 - 3a + q$

b) $5a^2 + 2ab + qb^2$

Practice: pg. ~~25~~



3.8B.4 Factoring Special Trinomials

Ex. Factor:

$$4x^2 - 9$$

$$a^2 - 25b^2$$

Difference of Squares:

Pattern can be applied to both multiplication of conjugates and factoring of difference of squares.

Ex. Multiply:

$$(3x - 4)(3x + 4)$$

$$(a^2 + 7b)(a^2 - 7b)$$

$$2(6q - 1)(6q + 1)$$

Ex. Factor:

$$x^2 - 81$$

$$-16c^2 + 25a^2$$

$$m^2 + 16$$

$$7g^3h^2 - 28g^5$$

Ex. Factor:

$$4x^2 + 12x + 9$$

$$x^2 - 8x + 16$$

Perfect Square Trinomial:

Ex. Multiply:

$$(x + 5)^2$$

$$(3q - 2)^2$$

$$3(2a + c)^2$$

How can we identify a perfect square trinomial?

Ex. Determine the values of q that would make each trinomial a perfect square:

$$x^2 + qx + 1$$

$$9x^2 + qx + 16$$

$$49a^2 + qab + 36b^2$$

Ex. Determine if each trinomial is a perfect square, if yes factor using the pattern:

$$x^2 + 6x + 9$$

$$c^2 - 14c + 36$$

$$25p^2 - 40pq + 16q^2$$

$$4x^2 + 10x + 25$$

Practice: pg. ~~246~~

Did You Hear About . . .

$(t+3)(t-2)$	STARTED
$(t+6)(t-1)$	WHO
$(t+6)(t-2)$	RED
$(t+5)(t-2)$	THE
$(t-9)(t+8)$	BECAUSE
$(t-4)(t+2)$	JOINED
$(t-10)(t+2)$	CROSS
$(t+7)(t-3)$	CAT
$(t+4)(t-3)$	AFTER
$(t-11)(t+1)$	THE

A	B	C	D
E	F	G	H
I	J	K	L
M	N	O	P
			?

Factor each trinomial below. Find the factored form in the answer column nearest the exercise, and notice the word beneath it. Write this word in the box containing the letter of that exercise. Keep working and you will hear about a kitty cat.

- (A) $t^2 + 3t - 10$
 - (B) $t^2 + 4t - 21$
 - (C) $t^2 + 5t - 6$
 - (D) $t^2 - 2t - 8$
 - (E) $t^2 - 10t - 11$
 - (F) $t^2 + 4t - 12$
 - (G) $t^2 - 8t - 20$
 - (H) $t^2 - t - 72$
-
- (I) $x^2 + 3x - 18$
 - (J) $x^2 - 17x - 18$
 - (K) $x^2 + 5x - 24$
 - (L) $x^2 - 10x - 24$
 - (M) $x^2 + 2xy - 15y^2$
 - (N) $x^2 - 5xy - 50y^2$
 - (O) $x^2 - 9xy - 36y^2$
 - (P) $x^2 + 5xy - 36y^2$

- (Q) $(x+6)(x-3)$
- (R) $(x+6)(x-4)$
- (S) $(x-25y)(x+2y)$
- (T) $(x-12)(x+2)$

When Is a Wrestler “King of the Ring”?

Factor each trinomial below. Find your answer and notice the letter next to it. Write this letter in the box containing the number of that exercise. Keep working and you will get the gripping answer to the title question.

- 1 $n^2 + 6n + 5$
- 2 $n^2 + 7n + 10$
- 3 $n^2 - 7n + 12$
- 4 $n^2 - 11n + 28$
- 5 $n^2 + 2n - 15$
- 6 $n^2 - 5n - 24$
- 7 $n^2 + n - 56$

- 8 $t^2 + 10t + 16$
- 9 $t^2 - 15t + 50$
- 10 $t^2 + 8t - 9$
- 11 $t^2 - 7t - 30$
- 12 $t^2 - t - 30$
- 13 $t^2 + 14t + 48$
- 14 $t^2 + 8t - 48$

- 15 $a^2 + 5ab + 6b^2$
- 16 $a^2 - 4ab - 21b^2$
- 17 $a^2 + 6ab - 7b^2$
- 18 $a^2 - 14ab - 32b^2$
- 19 $a^2 - 29ab + 100b^2$
- 20 $a^2 + 7ab - 18b^2$
- 21 $a^2 + 2ab + b^2$

Answers:

- L $(n + 2)(n + 6)$
- H $(n + 5)(n - 3)$
- W $(n + 5)(n + 1)$
- E $(n - 3)(n - 4)$
- B $(n - 1)(n + 15)$
- S $(n + 8)(n - 7)$
- H $(n + 2)(n + 5)$
- E $(n - 8)(n + 3)$
- R $(n - 12)(n - 2)$
- N $(n - 7)(n - 4)$

Answers:

- N $(t - 6)(t + 5)$
- V $(t - 25)(t + 2)$
- T $(t - 5)(t - 10)$
- T $(t + 6)(t + 8)$
- O $(t - 10)(t + 3)$
- B $(t + 15)(t - 2)$
- I $(t + 8)(t + 2)$
- H $(t - 4)(t + 12)$
- S $(t + 9)(t - 1)$
- A $(t - 24)(t + 2)$

Answers:

- K $(a - 8b)(a + 4b)$
- H $(a + 7b)(a - b)$
- A $(a - 20b)(a + 5b)$
- E $(a + 2b)(a + 3b)$
- W $(a + 9b)(a - 2b)$
- T $(a - 7b)(a + 3b)$
- O $(a - 25b)(a - 4b)$
- S $(a + 6b)(a + 3b)$
- N $(a + b)(a + b)$
- R $(a - 16b)(a + 2b)$

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Why Didn't Klutz Do Any Homework on Saturday?

Either multiply or factor, as directed, and find your answer in the adjacent answer column. Write the letter of that exercise in the box that contains the number of the answer.

Multiply:

- A $(a + 5)(a - 5)$
- B $(2 + 3a)(2 - 3a)$
- C $(7a - 1)(7a + 1)$
- D $(a^2 - 6)(a^2 + 6)$
- E $(4a + b)(4a - b)$
- F $(2a^2 - 5b)(2a^2 + 5b)$
- G $49a^2 - 1$
- H $a^2 - 25$
- I $4a^4 - 25b^2$
- J $4 - 9a^2$
- K $4a^4 - 36$
- L $a^4 - 36$

$$(4) \quad 16a^2 - b^2$$

Factor:

- M $x^2 - y^2$
- N $4x^2 - 49y^2$
- O $81x^2 - 100y^2$
- P $36x^2 - 121y^2$
- Q $9x^2 - 64y^2$
- R $x^4 - 400$
- S $9x + 10y)(9x - 10y)$
- T $(x + y)(x - y)$
- U $(x^2 + 20)(x^2 - 20)$
- V $(6x + 11y)(6x - 11y)$
- W $(3x + 7y)(3x - 7y)$
- X $(2x + 7y)(2x - 7y)$
- Y $(3x + 8y)(3x - 8y)$

$$(1) \quad (2n + 3)(2n - 3)$$

Factor:

- A $(12 + 5n)(12 - 5n)$
- B $(n + 1)(n - 1)$
- C $(7n + 3)(7n - 3)$
- D $(n + 7)(n - 7)$
- E $(9 + n)(9 - n)$
- F $(7n + 4)(7n - 4)$
- G $n^2 - 49$
- H $n^2 - 1$
- I $81 - n^2$
- J $4n^2 - 9$
- K $49n^2 - 16$
- L $144 - 25n^2$

Factor:

- M $a^6 - b^4$
- N $25a^8 - 9b^4$
- O $a^2b^2 - 36$
- P $16 - a^4b^6$
- Q $a^2b^4 - c^8$
- R $4a^{16} - 225$
- S $(4 + a^2b^3)(4 - a^2b^3)$
- T $(2a^8 + 15)(2a^8 - 15)$
- U $(a^3 + b^2)(a^3 - b^2)$
- V $(ab^2 + c^4)(ab^2 - c^4)$
- W $(ab + 6)(ab - 6)$
- X $(5a^4 + 3b^2)(5a^4 - 3b^2)$
- Y $(4 + ab^4)(4 - ab^4)$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
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OBJECTIVE 3-h: To simplify products of the form $(a + b)(a - b)$; to factor differences of squares.

What Happens If the Jolly Green Giant Steps on Your House?

For exercises in the first column, express each square as a trinomial. For the remaining exercises, factor each trinomial as the square of a binomial, if possible. (If this is not possible, the correct answer is "not possible.") Find your answer below. Write the letter of the exercise in the box containing the number of its answer.

Express as a trinomial:

- E $(u + 3)^2$
- O $(u - 8)^2$
- S $(2u + 5)^2$
- L $(1 - 4u)^2$
- T $(u + 2v)^2$
- U $(7u - 3v)^2$
- O $(uv + 6)^2$

Factor:

- E $t^2 + 4t + 4$
- U $t^2 - 12t + 36$
- L $t^2 - 18t + 81$
- Y $25 + 10t + t^2$
- W $4t^2 + 20t + 25$
- S $9t^2 - 12t + 4$
- I $t^2 + 10t + 20$

Factor:

- D $49a^2 + 14a + 1$
- O $16a^2 - 24a + 9$
- G $a^2 - 8a + 64$
- M $a^2 + 2ab + b^2$
- H $a^2 + 10ab + 25b^2$
- R $4a^2 - 12ab + 9b^2$
- M $100a^2 - 20ab + b^2$

Answers:

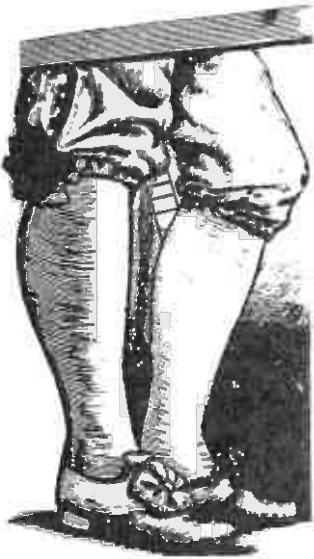
- 13 $4u^2 + 20u + 25$
- 3 $4u^2 + 16u + 25$
- 9 $u^2 + 6u + 9$
- 10 $u^2 + 4uv + 4v^2$
- 14 $49u^2 - 31uv + 9v^2$
- 6 $1 - 8u + 16u^2$
- 2 $u^2 - 16u + 64$
- 18 $u^2v^2 + 12uv + 36$
- 5 $u^2 + 7uv + 4v^2$
- 12 $49u^2 - 42uv + 9v^2$

Answers:

- 5 not possible
- 7 $(t - 9)^2$
- 19 $(t - 12)^2$
- 4 $(2t + 5)^2$
- 15 $(t + 2)^2$
- 21 $(3t - 2)^2$
- 16 $(2t - 9)^2$
- 3 $(t - 6)^2$
- 1 $(5 + t)^2$
- 8 $(3t - 5)^2$

Answers:

- 8 not possible
- 11 $(10a - 3b)^2$
- 16 $(7a + 1)^2$
- 11 $(10a - b)^2$
- 20 $(a + b)^2$
- 17 $(2a - 3b)^2$
- 19 $(4a - 3)^2$
- 20 $(a + 3b)^2$
- 14 $(a + 5b)^2$
- 19 $(4a - 8)^2$



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
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Why Does Gyro Never, Never, Ever Bet on Even Numbers?



Factor completely each polynomial below. Find your answer and notice the two letters next to it. Write these letters in the two boxes at the bottom of the page that contain the number of that exercise.

- 1 $3x^2 - 75$
- 2 $5x^2 + 30x + 45$
- 3 $x^3 - 49x$
- 4 $2x^2 - 24x + 72$

- 5 $2k^3 - 8k$
- 6 $54k^2 - 24$
- 7 $5k^3 + 100k^2 + 500k$
- 8 $12k^2 - 36k + 27$

- 9 $7a^3b - 7ab^3$
- 10 $32a^2b^2 + 16ab^2 + 2b^2$
- 11 $4a^3b - 40a^2b^2 + 100ab^3$
- 12 $4a^4b^3 - a^2b$

- 13 $5(x - 4)^2$
- 14 $2(x - 12)^2$
- 15 $3(x + 5)(x - 5)$
- 16 $x(x + 8)(x - 8)$

- 17 $5k(k + 10)^2$
- 18 $3(k - 2)^2$
- 19 $2k(k + 4)(k - 4)$
- 20 $6(3k + 2)(3k - 2)$

- 21 $5(x + 3)^2$
- 22 $2(x - 6)^2$
- 23 $3(x + 2)(x - 2)$
- 24 $x(x + 7)(x - 7)$

- 25 $2b^2(2a + 4)^2$
- 26 $4ab(a - 5b)^2$
- 27 $a^2b(2ab + 1)(2ab - 1)$
- 28 $7ab(a + b)(a - b)$

- 29 AT
- 30 AV
- 31 MA
- 32 IN

- 33 HE
- 34 LS
- 35 OR
- 36 TE

- 37 SF
- 38 NT
- 39 CH
- 40 ST

- 41 5
- 42 5
- 43 9
- 44 9
- 45 4
- 46 4
- 47 3
- 48 3
- 49 1
- 50 1
- 51 12
- 52 12
- 53 6
- 54 6
- 55 7
- 56 7
- 57 10
- 58 10
- 59 10
- 60 7
- 61 7
- 62 2
- 63 2
- 64 11
- 65 11
- 66 8
- 67 8

A DRASIC WAY TO DIET

AN EXTREME BUT EFFECTIVE WAY TO DIET IS HIDDEN IN THE LETTERS BELOW.
TO FIND IT:

Factor each trinomial below. Find the factored form in the set of answers under the exercise and cross out the letter above it. When you finish, the diet will remain. You might call it the "Algebra diet."



- 1 $m^2 + 8m + 7$
- 2 $m^2 + 5m + 6$
- 3 $m^2 + 10m + 9$
- 4 $m^2 - 6m + 8$
- 5 $m^2 - 8m + 12$
- 6 $m^2 + 11m + 24$

- 7 $d^2 - 8d + 15$
- 8 $d^2 - 12d + 20$
- 9 $d^2 + 14d + 13$
- 10 $d^2 - 13d + 36$
- 11 $d^2 + 17d + 30$
- 12 $d^2 + 9d + 18$

- 13 $x^2 + 5xy + 4y^2$
- 14 $x^2 - 18xy + 32y^2$
- 15 $x^2 - 13xy + 40y^2$
- 16 $x^2 + 7xy + 12y^2$
- 17 $x^2 - 27xy + 26y^2$
- 18 $x^2 + 19xy + 60y^2$

G	E	B	A	S	U	T	O	Y	F	N	U	L	E	O	M	A	T	O	R	E	G	I	A	N	L	T
(m - 2)(m - 4)	(m + 9)(m + 1)	(m + 8)(m + 1)	(m - 2)(m - 6)	(m + 7)(m + 1)	(m + 3)(m + 4)	(m + 2)(m + 3)	(m + 8)(m + 3)	(m - 2)(m - 8)	(d + 1)(d + 13)	(d + 2)(d + 9)	(d - 5)(d - 3)	(d - 10)(d - 2)	(d - 2)(d - 18)	(d - 5)(d - 4)	(d - 4)(d - 9)	(d + 6)(d + 3)	(x - 16y)(x - 2y)	(x + 4y)(x + 15y)	(x + 2y)(x + 4y)	(x + y)(x + 4y)	(x + 4y)(x + 3y)	(x + 20y)(x + 3y)	(x - 5y)(x - 8y)	(x - 2y)(x - 13y)	(x - 26y)(x - y)	
(m - 2)(m - 4)	(m + 9)(m + 1)	(m + 8)(m + 1)	(m - 2)(m - 6)	(m + 7)(m + 1)	(m + 3)(m + 4)	(m + 2)(m + 3)	(m + 8)(m + 3)	(m - 2)(m - 8)	(d + 1)(d + 13)	(d + 2)(d + 9)	(d - 5)(d - 3)	(d - 10)(d - 2)	(d - 2)(d - 18)	(d - 5)(d - 4)	(d - 4)(d - 9)	(d + 6)(d + 3)	(x - 16y)(x - 2y)	(x + 4y)(x + 15y)	(x + 2y)(x + 4y)	(x + y)(x + 4y)	(x + 4y)(x + 3y)	(x + 20y)(x + 3y)	(x - 5y)(x - 8y)	(x - 2y)(x - 13y)	(x - 26y)(x - y)	
(m - 2)(m - 4)	(m + 9)(m + 1)	(m + 8)(m + 1)	(m - 2)(m - 6)	(m + 7)(m + 1)	(m + 3)(m + 4)	(m + 2)(m + 3)	(m + 8)(m + 3)	(m - 2)(m - 8)	(d + 1)(d + 13)	(d + 2)(d + 9)	(d - 5)(d - 3)	(d - 10)(d - 2)	(d - 2)(d - 18)	(d - 5)(d - 4)	(d - 4)(d - 9)	(d + 6)(d + 3)	(x - 16y)(x - 2y)	(x + 4y)(x + 15y)	(x + 2y)(x + 4y)	(x + y)(x + 4y)	(x + 4y)(x + 3y)	(x + 20y)(x + 3y)	(x - 5y)(x - 8y)	(x - 2y)(x - 13y)	(x - 26y)(x - y)	

OBJECTIVE 3-I: To factor trinomials of the form $x^2 + bx + c$, where c is positive.