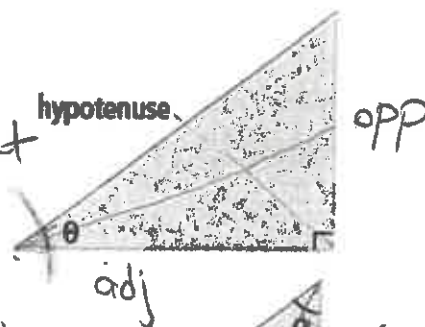


Unit 1: Right Angle Trigonometry1.1 The Tangent Ratio

θ "theta"
 α "alpha"

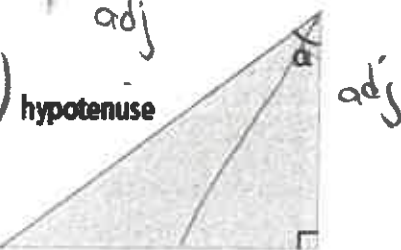
Right Triangle: A triangle with a 90° angle

Hypotenuse: the longest side of a right triangle
 - opposite the right angle.



Opposite Side:

The side across from the acute ($< 90^\circ$) angle being considered in a right Δ .



Adjacent Side: The side that forms one of the arms of the angle being considered. ^{opp}

(leg beside the angle which is NOT the hypotenuse)

A trigonometric ratio is a ratio of the measures of two sides of a right triangle.

Tangent Ratio: For any acute angle in a right Δ , the ratio of the length of the opposite side to the length of the adjacent side

$$\tan \theta = \frac{\text{length of the opposite side}}{\text{length of the adjacent side}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

Ex. Write a Tangent Ratio

Write each trigonometric ratio.

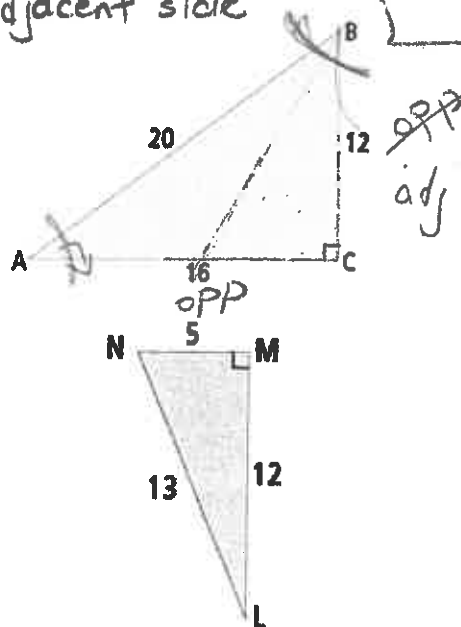
a) $\tan A = \frac{\text{opp}}{\text{adj}}$
 $= \frac{12}{16} \rightarrow \frac{3}{4}$

b) $\tan B = \frac{\text{opp}}{\text{adj}}$
 $= \frac{16}{12} = \frac{4}{3}$

Calculate each trigonometric ratio.

a) $\tan L = \frac{\text{opp}}{\text{adj}}$
 $= \frac{5}{12}$

b) $\tan N = \frac{\text{opp}}{\text{adj}}$
 $= \frac{12}{5}$

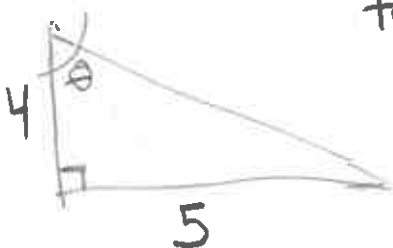


Ex. Calculate a Tangent and an Angle

a) Calculate $\tan 25^\circ$ to four decimal places.

$$\tan 25^\circ = 0.4663$$

b) Draw a triangle to represent $\tan \theta = \frac{5}{4}$. Calculate the angle θ to the nearest tenth of a degree.



$$\tan \theta = \frac{5 \leftarrow \text{opp}}{4 \leftarrow \text{adj}}$$

$$\tan^{-1}(\tan \theta) = \tan^{-1}\left(\frac{5}{4}\right)$$

$$\angle \theta = 51.3^\circ$$

Use your particular calculator to calculate each tangent ratio (to four decimal places) and angle (to the nearest degree).

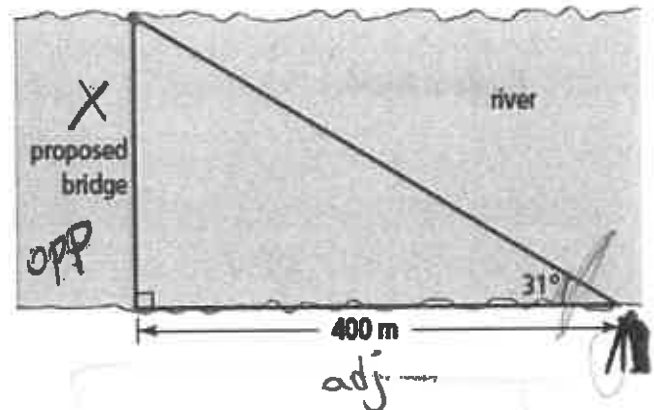
θ	Tan θ
27°	0.5095
45°	1
57°	1.5399

θ	Tan θ
27°	0.5095
29°	0.5543
56°	1.4653

$$\begin{aligned} \tan \theta &= 0.5095 \\ \theta &= \tan^{-1}(0.5095) \\ &= 27^\circ \end{aligned}$$

Ex. Determine a Distance Using the Tangent Ratio

A surveyor wants to determine the width of a river for a proposed bridge. The distance from the surveyor to the proposed bridge site is 400 m. The surveyor uses a theodolite to measure angles. The surveyor measures a 31° angle to the bridge site across the river. What is the width of the river, to the nearest metre?



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

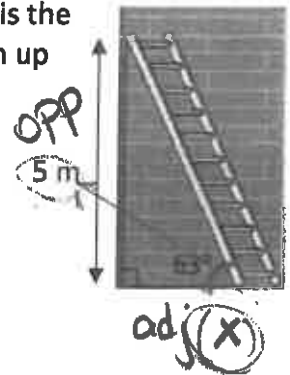
$$400 \cdot \tan 31^\circ = \frac{x}{400} \cdot 400$$

$$240\text{m} = x$$

$$4 \cdot \frac{x}{4} = 5.4$$

A ladder leaning against a wall forms an angle of 63° with the ground. How far is the foot of the ladder from the wall (to the nearest tenth) if the ladder reaches 5 m up the wall?

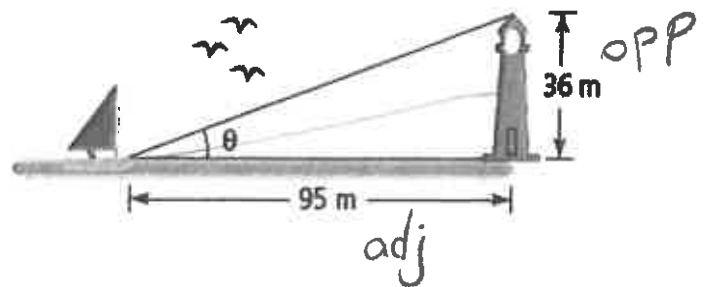
$$\begin{aligned}\tan \theta &= \frac{\text{opp}}{\text{adj}} \\ x \tan 63^\circ &= \frac{5}{x} \cdot x \\ x \cdot \tan 63^\circ &= 5 \\ \cancel{\tan 63^\circ} \quad \tan 63^\circ & \\ x &= 2.5 \text{ m}\end{aligned}$$



Ex. Determine an Angle Using the Tangent Ratio

A small boat is 95 m from the base of a lighthouse that has a height of 36 m above sea level. Calculate the angle from the boat to the top of the lighthouse. Express your answer to the nearest degree.

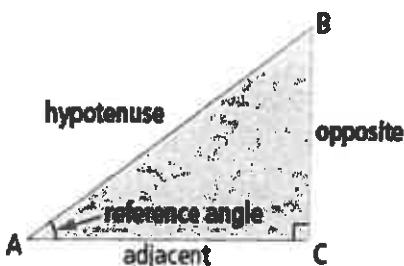
$$\begin{aligned}\tan \theta &= \frac{\text{opp}}{\text{adj}} \\ \tan \theta &= \frac{36}{95} \\ \theta &= \tan^{-1}(36 \div 95) \\ &= 21^\circ\end{aligned}$$



A radio transmission tower is to be supported by a guy wire. The wire reaches 30 m up the tower and is attached to the ground a horizontal distance of 14 m from the base of the tower. What angle does the guy wire form with the ground, to the nearest tenth of a degree?



$$\begin{aligned}\tan \theta &= \frac{\text{opp}}{\text{adj}} \\ \tan \theta &= \frac{30}{14} \\ \theta &= \tan^{-1}(30 \div 14) \\ &= 64.98^\circ \\ &= 65.0^\circ\end{aligned}$$

1.2 Sine & Cosine Ratios

Sine and cosine also use right Δ s.

$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

SOH
CAH
TOA

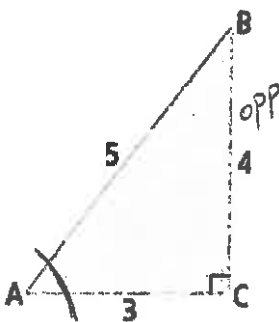
Ex. Write Trigonometric Ratios

Write each trigonometric ratio.

a) $\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$

b) $\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$

c) $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$



d) $\sin B = \frac{3}{5}$

e) $\cos B = \frac{4}{5}$

f) $\tan B = \frac{3}{4}$

Ex.

a) Evaluate each ratio, to four decimal places.

$$\sin 42^\circ = 0.6691$$

$$\cos 68^\circ = 0.3746$$

b) Determine each angle measure, to the nearest degree.

$\beta = \text{'beta'}$

\sin^{-1}
 $\sin \theta = 0.4771$

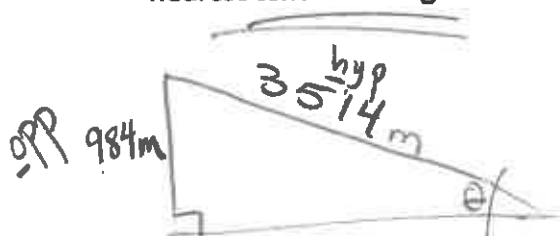
$$\theta = \sin^{-1}(0.4771) = 28^\circ$$

$\cos \beta = 0.7225$

$$\beta = \cos^{-1}(0.7225) = 44^\circ$$

SOH CAH TOA

Ex. In the World Cup Downhill held at Panorama Mountain Village in British Columbia, the skiers raced 3514 m down the mountain. If the vertical height of the course was 984 m, determine the average angle of the ski course with the ground. Express your answer to the nearest tenth of a degree.



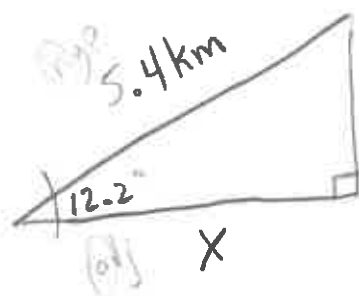
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin \theta = \frac{984}{3514}$$

$$\theta = \sin^{-1}(984 \div 3514)$$

$$= 16.3^\circ$$

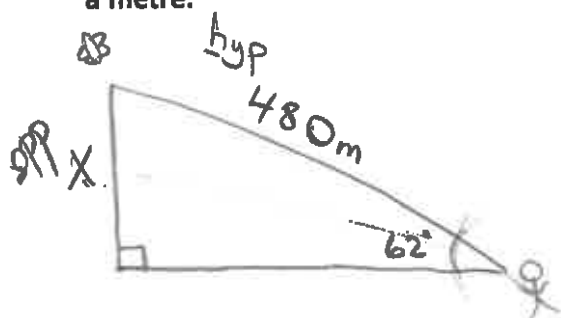
Ex. A pilot starts his takeoff and climbs steadily at an angle of 12.2° . Determine the horizontal distance the plane has travelled when it has climbed 5.4 km along its flight path. Express your answer to the nearest tenth of a kilometre.



$$5.4 \cos 12.2^\circ = \frac{x}{5.4} \cdot 5.4$$

$$5.3 \text{ km} = x$$

Ex. Determine the height of a kite above the ground if the kite string extends 480 m from the ground and makes an angle of 62° with the ground. Express your answer to the nearest tenth of a metre.



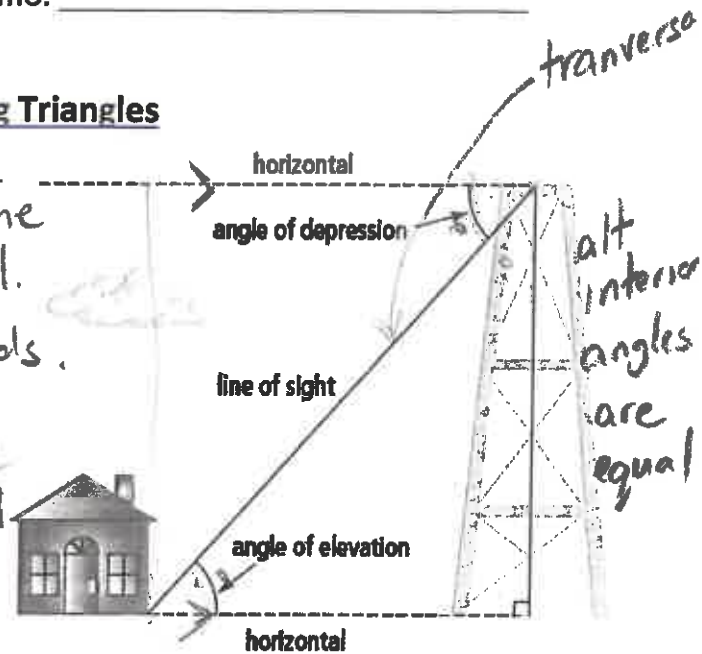
$$480 \sin 62^\circ = \frac{x}{480} \cdot 480$$

$$423.8 \text{ m} = x$$

2.3 Problems & Solving Triangles

Angle of Elevation: The angle b/w the line of sight and the horizontal when an observer looks upwards.

Angle of Depression: The angle b/w the line of sight and the horizontal when the observer looks downwards.



Ex. Sean wants to calculate the height of the First Nations Native Totem Pole. He positions his transit 19.0 m to the side of the totem pole and records an angle of elevation of 63° to the top of the totem pole. If the height of Sean's transit is 1.7 m, what is the height of the totem pole, to the nearest tenth of a metre?

SOH CAH TOA

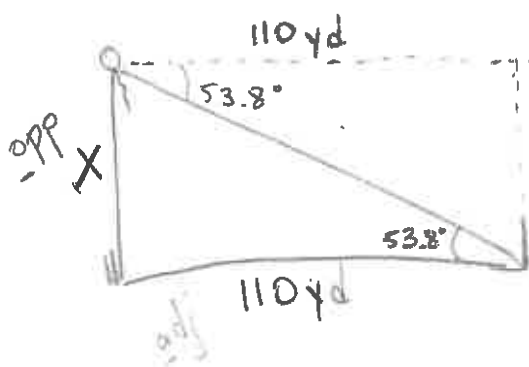
$$19 \cdot \tan 63^\circ = \frac{x}{19}$$

$$37.3 = x$$

$$+ 1.7 \leftarrow \text{transit}$$

$$\boxed{39.0 \text{ m}} \leftarrow \text{totem pole height}$$

Ex. A balloonist decides to use an empty football field for his landing area. When the balloon is directly over the goal post, he measures the angle of depression to the base of the other goal post to be 53.8° . Given that the distance between goal posts in a Canadian football field is 110 yd, determine the height of the balloon.

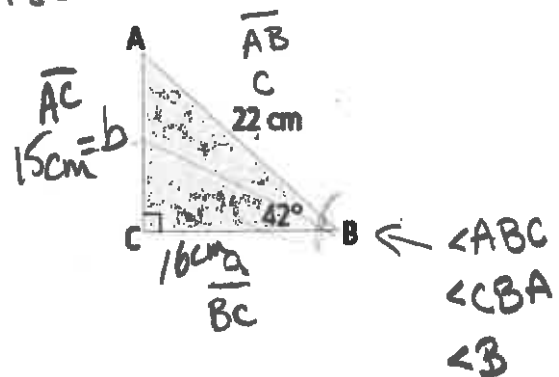


$$110 \cdot \tan 53.8^\circ = \frac{x}{110}$$

$$\boxed{150.3 \text{ yd} = x}$$

Solve a Triangle: \rightarrow 6 pieces of info \rightarrow 3 sides, 3 angles
 SOH CAH TOA, Pythagoras, $\text{sum } \angle s = 180^\circ$

Ex. Solve the triangle shown. Express each measurement to the nearest whole unit.



$$22 = \sin 42^\circ = \frac{b}{22} \cdot 22$$

$$15 \text{ cm} = b$$

$$\angle A =$$

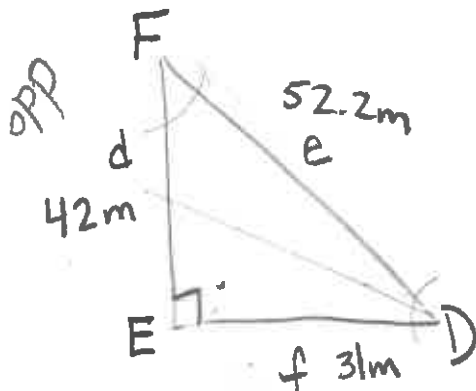
$$\angle A = 180 - 90 - 42$$

$$\angle A = 48^\circ$$

$$22 \cdot \cos 42 = \frac{a}{22} \cdot 22$$

$$16 \text{ cm} = a$$

Ex. Solve $\triangle DEF$ if $\angle E = 90^\circ$, $d = 42 \text{ m}$, and $f = 31 \text{ m}$. Express sides to the nearest tenth & angles to the nearest degree.



$$42^2 + 31^2 = e^2$$

$$1764 + 961 = e^2$$

$$\sqrt{2725} = \sqrt{e^2}$$

$$52.2 \text{ m} = e$$

$$\angle D$$

$$\tan D = \frac{42}{31}$$

$$\angle D = \tan^{-1}(42 \div 31)$$

$$\angle D = 54^\circ$$

$$\angle F$$

$$\angle F = 180 - 90 - 54$$

$$\angle F = 36^\circ$$