

Unit 6Unit 8: Solving Systems of Linear Equations Graphically (Ch 8)6.8.1 Systems of Linear Equations & Graphs

**Recall:** To **SOLVE** an equation means to identify the value of the variable which satisfies the equation (makes the equation true).

Ex. Solve  $2x - 1 = 9$

$$\begin{array}{r} +1 \quad +1 \\ \hline x = 5 \end{array}$$

$$2x = 10$$

$$\text{Check } 2x - 1 = 9$$

$$2(5) - 1 = 9$$

$$10 - 1 = 9$$

$$9 = 9$$

**System of Linear Equations:** Two or more linear equations involving common variables

\* Number of equations equals the number of variables.

Ex. Solve the system:  $y = 2x$  and  $y = x + 2$

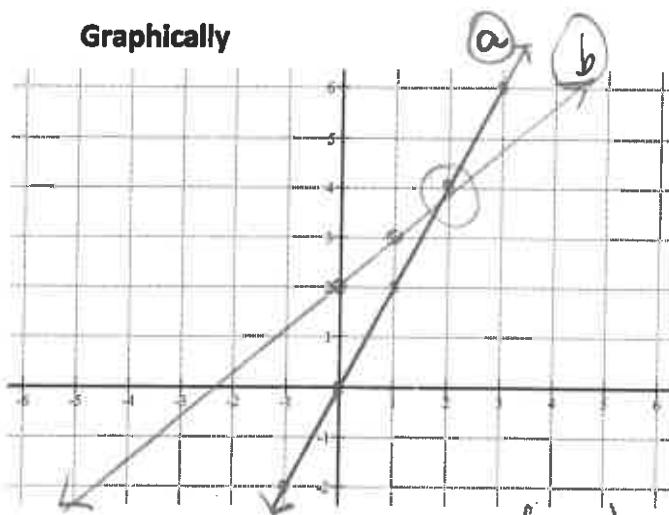
**Numerically**

$$y = 2x + 0 \quad y = x + 2$$

x	y
0	0
1	2
2	4
3	6
4	8

x	y
0	2
1	3
2	4
3	5
4	6

**Graphically**



Point of intersection

is the solution

(2, 4)

**Algebraic Verification**

$$\begin{aligned} y &= 2x & y &= x + 2 \\ 4 &= 2(2) & 4 &= 2 + 2 \\ 4 &= 4 & 4 &= 4 \end{aligned}$$

**Solution (to a system of linear equations):**

- 1) a pair of values occurring in the table of values of both equations  $x = 2$ ,  $y = 4$
- 2) a point of intersection of the lines on a graph
- 3) an ordered pair that satisfies both equations

**Ex.** Consider the system of linear equations  $2x + y = 2$  and  $x - y = 7$ . Identify the solution of the system by graphing, then verify the solution.

$$\textcircled{a} \quad 2x + y = 2$$

$$\begin{array}{l} \text{y-int} \\ \hline y=0 \end{array} \quad 2x + (0) = 2 \quad (1, 0)$$

$$2x = 2$$

$$x = 1$$

$$\begin{array}{l} \text{y-int} \\ \hline x=0 \end{array} \quad 2(0) + y = 2 \quad (0, 2)$$

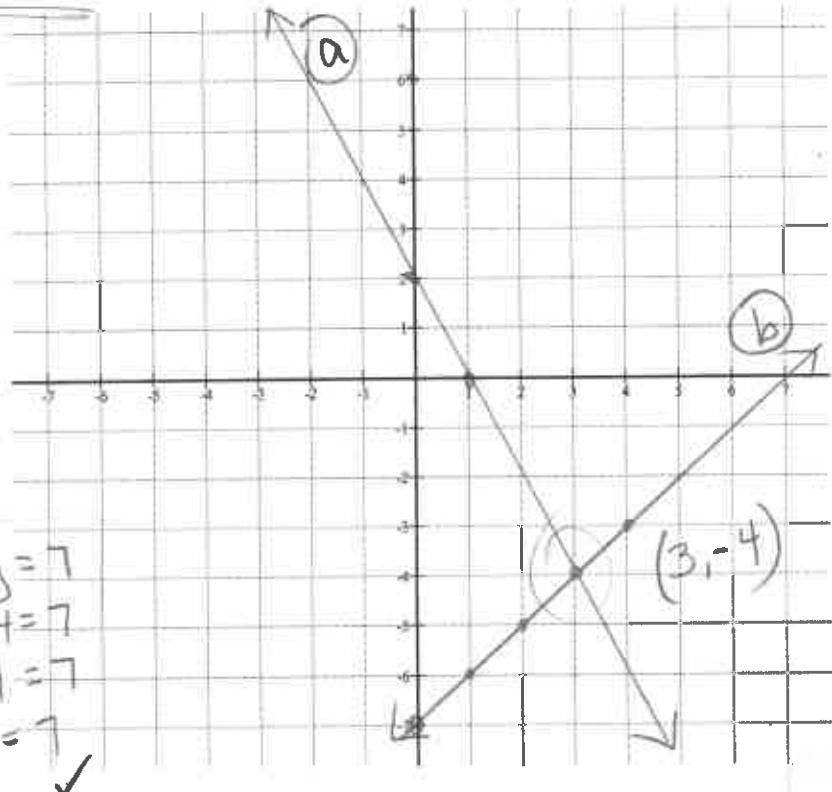
$$y = 2 \quad \underline{\text{Check}}$$

$$x - y = 7$$

$$\begin{array}{l} x-y=7 \\ 2x+y=2 \\ \hline 3x+0=9 \\ 3x=9 \\ x=3 \end{array}$$

$$\textcircled{b} \quad \begin{array}{l} x-7=y \\ 1x-7=y \end{array}$$

$$\begin{array}{l} 2(3)+(-4)=2 \\ 6-4=2 \\ 2=2 \end{array}$$



**Ex.** Guy solved the linear system  $x - 2y = 12$  and  $3x - 2y = 4$ . His solution is  $(2, -5)$ . Verify whether Guy's solution is correct. Explain how Guy's results can be illustrated on a graph.

Check

$$x - 2y = 12$$

$$2 - 2(-5) = 12$$

$$2 + 10 = 12$$

$$12 = 12 \quad \checkmark$$

$$3x - 2y = 4$$

$$3(2) - 2(-5) = 4$$

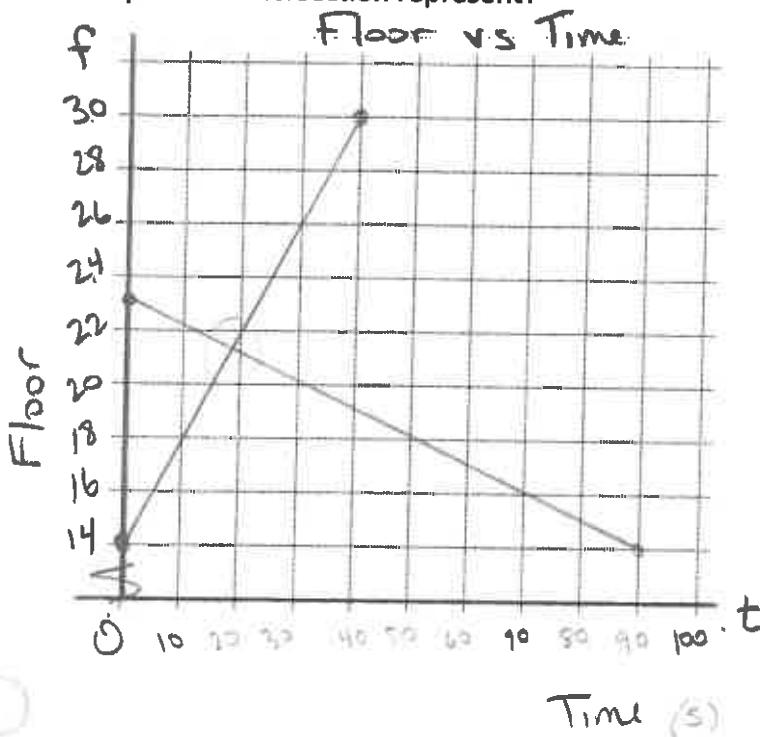
$$6 + 10 = 4$$

$$16 = 4 \quad \times$$

His solution is NOT correct.

The point  $(2, -5)$  is only on one of the two lines.

Ex. Eric works on the 23<sup>rd</sup> floor of a building. It takes Eric 90 s to walk down the stairs to the 14<sup>th</sup> floor. Nathan works on the 14<sup>th</sup> floor and can get to the 30<sup>th</sup> floor by elevator in 40 s. Suppose both men leave their offices at the same time. Create a graph to model their travel. What does the point of intersection represent?



Eric     $t=0$   
23<sup>rd</sup> floor  
 $(0, 23)$

$t=90$   
14<sup>th</sup> floor  
 $(90, 14)$

Nathan  
 $(0, 14)$

$(40, 30)$

19s 21<sup>st</sup> floor

At 19s they are  
the 21<sup>st</sup> floor

Practice: pg. 427/ # 1, 3, 5 – 7, 9 – 11, 17, 19



6.3.2 Modelling & Solving Linear Systems

**Ex.** Translate each description into an algebraic expression. Define your variable.

- a) Double a boat's speed increased by 3 km/h

let  $b$  = boat's speed

$$2b + 3$$

- b) \$7 less than the ticket price

let  $t$  = ticket price

$$t - 7$$

- c) Triple a number decreased by half the number

Let  $x$  = the number

$$3x - \frac{1}{2}x$$

**Ex.** People can rent ski and snowboard equipment from two places at Winterland Resort.

Option A charges a one-time \$30 fee and then \$8 per hour.

Option B charges \$14 per hour.

Let  $C$  = cost (total)

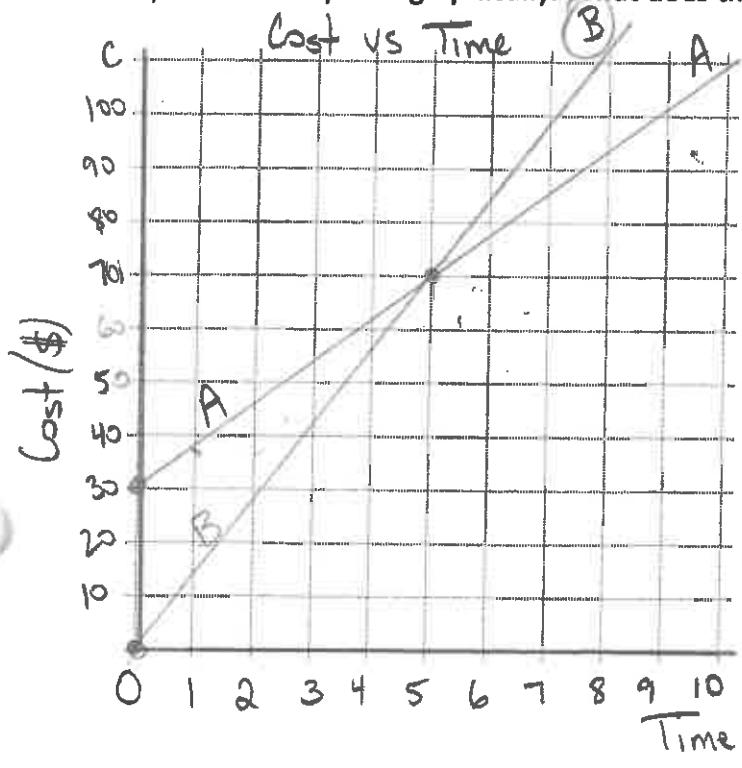
- a) Create a system of linear equations to model the rental charges.

$$(A) C = 8t + 30$$

$$(B) C = 14t$$

$t$  = time rented · (x)

- b) Solve the system graphically. What does the solution represent?



$$\textcircled{A} \quad C = 8t + 30$$

$$\frac{8}{1} \times 5 = \frac{40}{5}$$

$$\textcircled{B} \quad C = 14t$$

$$\frac{14}{1} \times 5 = \frac{70}{5}$$

Solution (5, 70)

The plan costs the same amount at \$70 when renting for 5 hours.  
 'A' is better if renting for more than 5 hours.  
 'B' is better if renting for less than 5 hours.

**Ex.** Two hopper-bottom grain bins are being emptied starting at the same time.

- The larger bin holds  $45 \text{ m}^3$  of grain. It is emptied at a rate of  $1 \text{ m}^3$  per minute.
- The smaller bin stores  $30 \text{ m}^3$  of grain. This bin is emptied at a rate of  $0.5 \text{ m}^3$  per minute.

- a) Model the volume of grain remaining as a function of time using a system of linear equations.

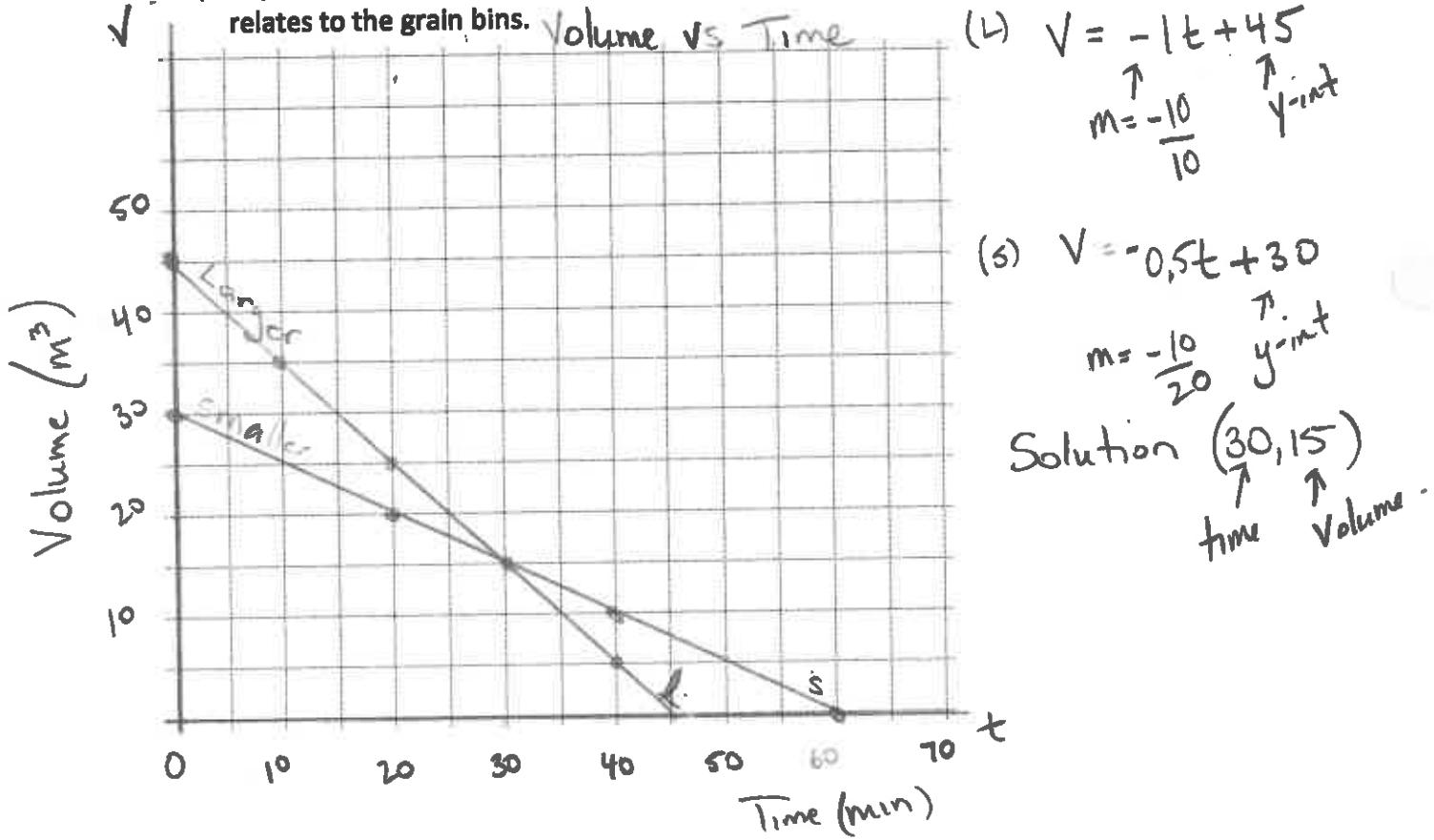
Let  $t = \text{time (min)}$

Let  $V = \text{volume of grain } (\text{m}^3)$

$$(L) V = 45 - 1t$$

$$(S) V = 30 - 0.5t$$

- b) Represent the linear system graphically. Describe how the information shown in the graph relates to the grain bins.



At 30 min both bins have  $15 \text{ m}^3$  of grain. Before 30 min, the larger bin contains more grain. After 30 min, the smaller bin contains more grain.

Practice: pg. 440 / # 1 – 6, 8, 11, 18, 19, 24

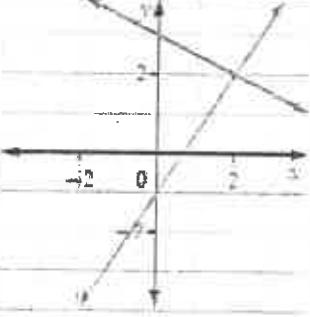
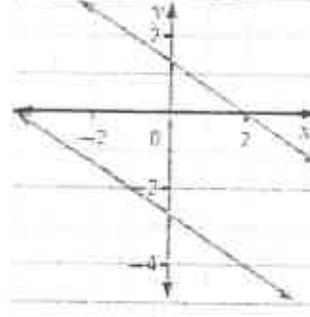
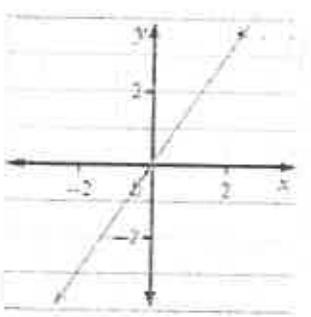
6.3 Number of Solutions for Systems of Linear Equations

In terms of solutions, there are 3 possibilities when solving a system of linear equations:

1. one solution
2. zero solution
3. infinite solutions

Before solving, you can predict the number of solutions by comparing the slopes and the y-intercepts of the equations.

*same line on top of itself-*

	Intersecting Lines	Parallel Lines	Coincident Lines
Number of Solutions	1	0	infinite
Graph			
Slopes:	different	same	same
y-intercepts:	different or same	different	same

$$y = 3x - 5$$

$$y = 2x - 5$$

**Ex.** Predict the number of solutions for each system of linear equations. Explain your reasoning, and then confirm each answer by graphing the linear system.

$$\text{a) } y = 2x - 3$$
$$y = \frac{1}{2}x + 3$$

m ≠

## 1 Solution

$$\begin{array}{l} \text{(1)} 4x + 10y = 30 \\ \text{(2)} 2x + 5y = 35 \end{array}$$

10-2-11-10

$$\frac{10y}{10} = -\frac{4x}{10} + \frac{30}{10} \quad \frac{5y}{5} = -2x + 35$$

$$\textcircled{1} \quad y = -\frac{2}{3}x + 3$$

$$\frac{5y}{5} = -2x + \frac{35}{5}$$

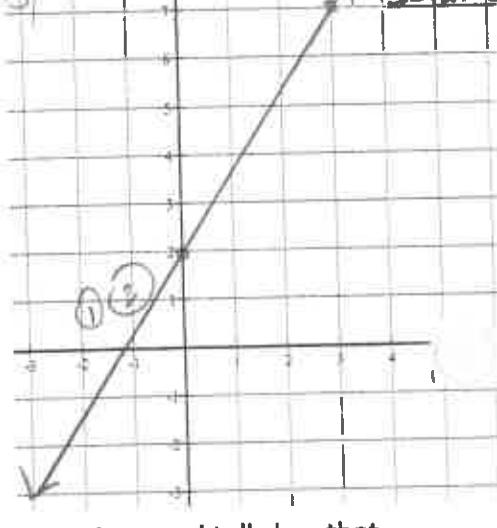
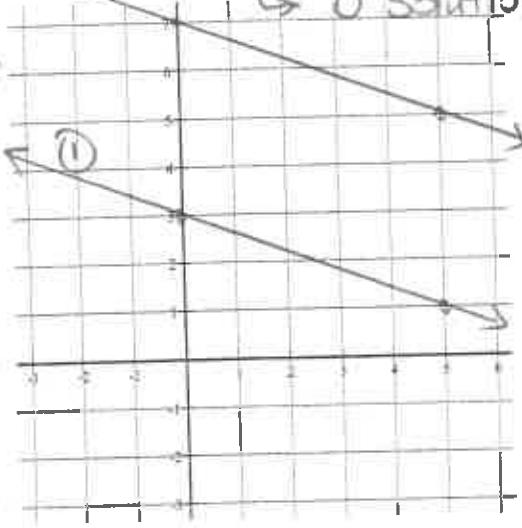
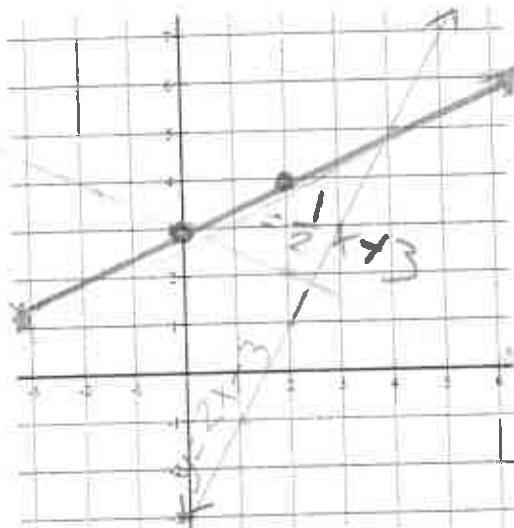
$$\text{c) } \textcircled{1} \quad 10x - 6y = \underline{\underline{-12}}$$

$$3 \frac{21y}{51} = \frac{42 + 35x}{51} \rightarrow y = 2 + \frac{5}{3}x$$

$$y = -\frac{2}{3}x + 7 \quad ① \frac{5}{3}x + 2 = y$$

51

$m = , b = \Rightarrow$  same line  
infinite solutions



**Example 2:** Sabrina's teacher gives her the following systems of linear equations and tells her that each system has either no solution or an infinite number of solutions. How can Sabrina determine each answer by inspecting the equations?

$$\begin{aligned} \text{a) } 2x + 3y &= 12 \\ -2x + 3y &= 20 \end{aligned}$$

$$2x + 3y = 20$$

$$\begin{matrix} n = \\ b \neq \end{matrix} \left. \right\} \text{parallel}$$

$\Rightarrow$  No solution

b)  $2x + 3y = 12$

$$\frac{4x}{2} + \frac{6y}{2} = \frac{24}{2} \quad 2x + 3y = 12$$

Reduce  $\rightarrow$  lowest terms

eqns are the same

infinite solution