FMP10 Name_____

Exponent Law Practice

Exponent Law	
Note that a and b are rational or variable bases and m and n are integral exponents.	
Product of Powers	$(a^m)(a^n) = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = (a^m)(b^m)$
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
Zero Exponent	$a^0 = 1, a \neq 0$

Simplify, then evaluate.

1. $\frac{4^5 \times 4^6}{4^3} =$

2. $-b^0 =$

3. $(3^2)^3 =$

4. $\frac{(5+3)^2}{8^5} =$

5. $\left(\frac{2}{3}\right)^{-4} =$

6. $(-4)^0 =$

7. $\frac{7a^2b^6c^3}{35a^3b^2c} =$

8. $(3^2)^4 \times (2^3)^2 =$

9. $\left(\frac{-3}{x^{-4}}\right)$

FMP10 Name_____

Exponent Law Practice

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9. $\left(\frac{-3}{x^{-4}}\right)$

2.0 LCM, GCF, Prime Factorization

FMP10 Name: _____

Date: _____

Unit 2: Exponents & Radicals

2.1 Square & Cube Roots

Perfect Square:

Square Root:

Perfect Cube:

Cube Root:

Ex.

Determine the value of each expression. Express your answer as integers or fractions in lowest terms.

a) 6^3 b) $(-4)^2$ c) -4^2 d) $\frac{4^3}{6}$

Prime factorization (factor tree) can be used to determine if a number is a perfect square or cube.

Ex.

Determine and state if each number is a perfect square, perfect cube, both, or neither:

a) 121 b) 729

c) 356 d) 4096

Ex.

Evaluate, with the aid of a calculator if necessary:

a) $\sqrt{324}$ b) $\sqrt[3]{13824}$ c) $\sqrt{\frac{25}{361}}$ d) $\sqrt[3]{175616}$

e) $\sqrt{36x^2}$ f) $\sqrt[3]{8q^3}$ g) $\sqrt{961z^6}$ h) $\sqrt[3]{512m^{12}}$

i) $\sqrt[3]{\frac{27x^9}{1000y^{15}}}$ k) $\sqrt{\frac{100q^{10}u^{12}}{169x^{18}}}$

Ex.

a) A floor mat for gymnastics is a square with an area of 196 m^2. What is its side length?

b) The volume of a cubic box is 27 000 $in.^3$ Determine its side length.

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2.2 Integral Exponents

Exponent Law Summary

Exponent Law	
Note that a and b are rational or variable bases and m and n are integral exponents.	
Product of Powers	$(a^m)(a^n) = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = (a^m)(b^m)$
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
Zero Exponent	$a^0 = 1, a \neq 0$

Exploring Negative Exponents:

Continue the pattern:

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 =$

$2^0 =$

$2^{-1} =$

$2^{-2} =$

$2^{-3} =$

$2^{-4} =$

$3^4 = 81$

$3^3 = 27$

$3^2 = 9$

$3^1 =$

$3^0 =$

$3^{-1} =$

$3^{-2} =$

$3^{-3} =$

$3^{-4} =$

Simplify using expansion and cancellation, then simplify by exponent laws:

$\frac{4^2}{4^5}$

$\frac{(-2)^3}{(-2)^7}$

Summary:

$a^{-n} =$

$\frac{1}{a^{-n}} =$

$\left(\frac{a}{b}\right)^{-n} =$

Ex. Write each expression with positive exponents.

5^{-2}	$(4y)^{-3}$
x^{-5}	$4y^{-3}$
$\left(\frac{4}{5}\right)^{-3}$	$\frac{1}{6^{-3}}$
xy^{-2}	$\frac{1}{z^{-5}}$
$x^3y^{-1}z^{-7}$	$(xy)^{-2}$

Ex. Write each product or quotient as a power with a single exponent.

$(5^8)(5^{-3})$ $(0.8^{-2})(0.8^{-4})$ $\dfrac{(2x)^3}{(2x)^{-2}}$

$(-4)^3(-4)^{-8}$ $\dfrac{x^5}{x^{-3}}$ $\dfrac{q^{-1}}{q^7}$

Ex. Write each expression as a power with a single, positive exponent. Then, evaluate where possible.

$(4^3)^{-2}$ $\left(\dfrac{2^4}{2^6}\right)^{-3}$ $\left[\dfrac{(y^2)^0}{(y^{-4})^2}\right]^{-3}$

$[(a^{-2})(a^3)]^{-1}$ $\left[\left(\dfrac{3}{4}\right)^{-2}\left(\dfrac{3}{4}\right)^4\right]^{-2}$

Ex. It is estimated that there are 108 billion grasshoppers in an area of 27 000 km^2 of Saskatchewan. Approximately how many grasshoppers are there per square kilometre? Solve arithmetically and by using exponent laws.

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2.3 Rational Exponents - Extension

Ex. Write each product or quotient as a power with a single positive rational exponent.

a) $\left(5^{\frac{1}{3}}\right)\left(5^{\frac{5}{3}}\right)$ b) $(x^5)\left(x^{-\frac{1}{2}}\right)$ c) $\frac{3^{-\frac{3}{4}}}{3^{0.25}}$ d) $\frac{8^{1.8}}{16^{0.3}}$

Ex. Write each expression as a power with a single positive rational exponent. Then, evaluate where possible.

a) $(x^{1.5})(x^{3.5})$ b) $\left(p^{\frac{-5}{4}}\right)\left(p^{\frac{1}{2}}\right)$ c) $\frac{4^{\frac{1}{2}}}{4^{0.5}}$ d) $\frac{1.5^{\frac{4}{3}}}{1.5^{\frac{1}{6}}}$

We can use our exponent laws to deal with rational exponents, but what do they mean?

$$\left(25^{\frac{1}{2}}\right)\left(25^{\frac{1}{2}}\right) =$$

What else when multiplied by itself gives a product of 25? _____

What is the relationship between $25^{\frac{1}{2}}$ and 5?

Ex. Write each expression as a power with a single positive rational exponent. Then, evaluate where possible.

a) $(4x^3)^{0.5}$ b) $\left[(x^3)\left(x^{\frac{3}{2}}\right)\right]^{\frac{1}{2}}$ c) $\left(\frac{3^4}{16}\right)^{-0.75}$

Ex. Simplify and evaluate where possible.

a) $(27x^6)^{\frac{2}{3}}$ b) $\left[\left(x^{\frac{4}{3}}\right)\left(x^{\frac{1}{3}}\right)\right]^9$ c) $\left(\frac{x^3}{64}\right)^{-\frac{2}{3}}$

Ex. Food manufacturers use a beneficial bacterium called *Lactobacillus bulgaricus* to make yogurt and cheese. The growth of 10 000 bacteria can be modeled using the formula $N = 10\,000(2)^{\frac{h}{42}}$, where *N* is the number of bacterial after *h* hours.

a) What does the value of 2 in the formula tell you?

b) How many bacteria are present after 42 h?

c) How many more bacteria are present after 2 h?

d) How many bacteria are present after 105 h?

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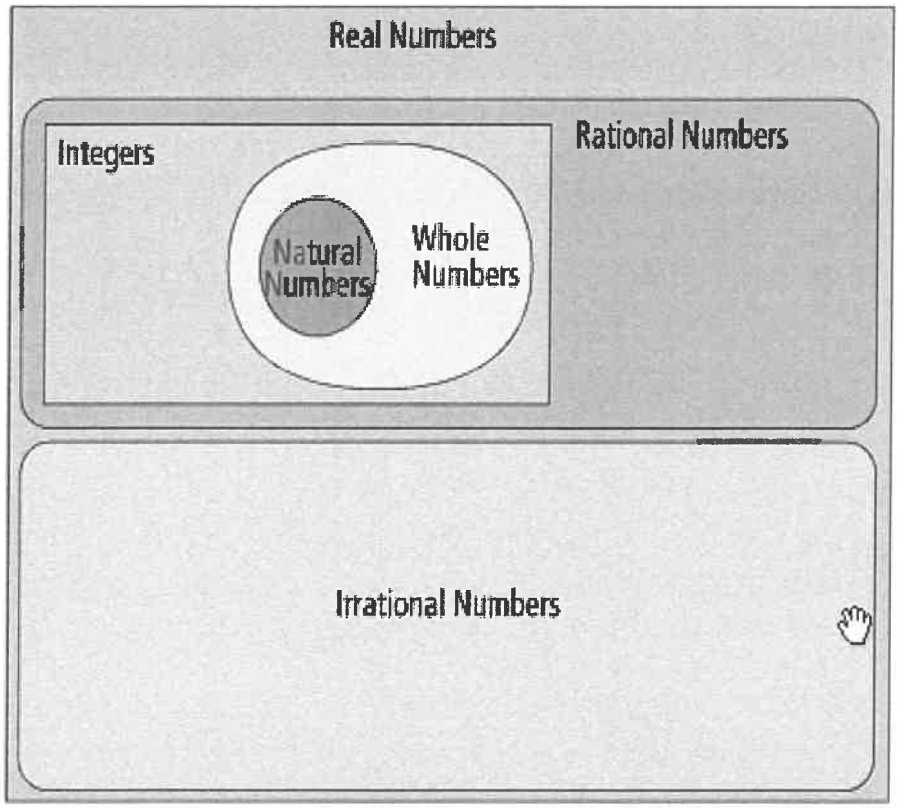
2.4A

2.4A Irrational Numbers I Extension

Irrational Numbers:

Number Sets:

Place the letter corresponding with the correct description in the appropriate area on the diagram.



A.
Examples include π & $\sqrt{2}$

B.
Examples include
$0.3, \frac{4}{5}, \& \sqrt{25}$

C.
1, 2, 3,...

D.
...-3, -2, -1, 0, 1, 2, 3...

E.
0, 1, 2, 3,...

F.
All rational & irrational numbers

Label: $\sqrt[n]{x}$

Rational Exponent Law:

Ex. Express each power as an equivalent radical.

a) $64^{\frac{1}{2}}$

b) $16^{\frac{3}{4}}$

c) $(8x^2)^{\frac{1}{3}}$

d) $10^{\frac{1}{4}}$

e) $1024^{\frac{1}{3}}$

f) $(x^4)^{\frac{3}{8}}$

Ex. Express each radical as a power with a rational exponent.

a) $\sqrt[4]{5^3}$

b) $\sqrt[5]{3^4}$

c) $\left(\sqrt{s^3}\right)$

d) $\sqrt{125}$

e) $\sqrt[3]{y^5}$

f) $\left(\sqrt[n]{27^2}\right)$

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2.4B

~~4.4B~~ Irrational Numbers II - Extension

Mixed radical:

Entire radical:

Ex. Express each mixed radical as an equivalent entire radical.

a) $5\sqrt{11}$ b) $2\sqrt[3]{5}$ c) $5\sqrt[3]{6}$

Ex. Express each entire radical as an equivalent mixed radical.

a) $\sqrt{27}$ b) $\sqrt{50}$ c) $\sqrt{48}$ d) $\sqrt[3]{32}$

e) $\sqrt{40}$ f) $\sqrt{108}$ g) $\sqrt[3]{54}$ h) $\sqrt[4]{80}$

Ex. Order these irrational numbers from least to greatest without a calculator.

$2\sqrt{18}$ $\sqrt{8}$ $3\sqrt{2}$ $\sqrt{32}$

Ex. Order these numbers from least to greatest. Identify which numbers are irrational.

$\sqrt{5}$ $\frac{5}{2}$ $2.\overline{2}$ $2\sqrt{2}$ $\sqrt[3]{8}$

Ex. The Seabee Mine is located in Laonil Lake, SK. In 2007, the mine produced a daily average of gold great enough to fill a cube with a volume of 180 cm^3. If five days of gold production is cast into a cube, what is its edge length?

Practice 2.4A

1. Express each power as an equivalent radical.

a) $5^{\frac{3}{2}}$

b) $\left(27^2\right)^{\frac{2}{3}}$

c) $(4x^3)^{0.5}$

d) $\left(\frac{x^4}{y^2}\right)^{\frac{-3}{2}}$

e) $\left(x^6y\right)^{\frac{1}{3}}$

2. Express each radical as a power.

a) $\sqrt{(9x)^3}$

b) $\sqrt{\left(4x^2\right)^3}$

c) $\sqrt[3]{64x^6}$

d) $\sqrt[4]{x^0y^2}$

e) $9\sqrt[5]{x^{\frac{5}{2}}}$

3. Evaluate each expression. Give the result to four decimal places, if necessary.

a) $14^{\frac{3}{2}}$

b) $5(0.8)^{\frac{1}{3}}$

c) $\frac{\sqrt{9}}{\sqrt{12}}$

d) $\sqrt[3]{25}$

1. a) $\sqrt{5^3}$ **b)** $\left(\sqrt[3]{27}\right)^4$ **c)** $\sqrt{4x^3}$ **d)** $\sqrt{\frac{y^6}{x^{12}}}$ **e)** $\sqrt[3]{x^6y}$ **2. a)** $(9x)^{\frac{3}{2}}$ **b)** $\left(4x^2\right)^{\frac{3}{2}}$ **c)** $\left(64x^6\right)^{\frac{1}{3}}$ **d)** $y^{\frac{1}{2}}$ **e)** $9x^{\frac{1}{2}}$

3. a) 52.3832 **b)** 4.6416 **c)** 0.8660 **d)** 2.9240

Practice 2.4B

1. Express each mixed radical as an equivalent entire radical.

a) $5\sqrt{3}$

b) $\left(\frac{2}{5}\right)\sqrt{10}$

c) $2\sqrt[3]{4}$

d) $-4\sqrt[3]{2}$

e) $5\sqrt[3]{3}$

2. Express each entire radical as an equivalent mixed radical.

a) $\sqrt{180}$

b) $\sqrt{108}$

c) $\sqrt[3]{750}$

d) $\sqrt[3]{81}$

e) $\sqrt{486}$

3. Order each set of numbers from greatest to least. Describe the method you used.

a) $\sqrt{35}, \sqrt{\frac{5}{3}}, \sqrt[3]{45}, 3\sqrt{20}$

b) $4\sqrt{5},\ 2\sqrt[3]{5}, \sqrt{60}, \sqrt[3]{4}$

1. a) $\sqrt{75}$ **b)** $\sqrt{\frac{8}{5}}$ **c)** $\sqrt[3]{32}$ **d)** $\sqrt[3]{-128}$ **e)** $\sqrt[3]{375}$ **2. a)** $6\sqrt{5}$ **b)** $6\sqrt{3}$ **c)** $5\sqrt[3]{6}$ **d)** $3\sqrt[3]{3}$ **e)** $9\sqrt{6}$

3. a) $3\sqrt{20}, \sqrt{35}, \sqrt[3]{45}, \sqrt{\frac{5}{3}}$. Example: I estimated the values and plotted the values on a number line.

b) $4\sqrt{5}, \sqrt{60}, 2\sqrt[3]{5}, \sqrt[3]{4}$. Example: I converted each mixed radical to an entire radical.

2.1

Extra Practice

1. Determine whether each of the following numbers is a perfect square, a perfect cube, both, or neither. Justify your choices mathematically.
   a) 196
   b) 200
   c) 343
   d) 625
   e) 729
   f) 3375

2. Evaluate using prime factorization.
   a) $\sqrt{256}$
   b) $\sqrt{225}$
   c) $\sqrt[3]{1000}$
   d) $\sqrt{1681}$
   e) $\sqrt[3]{512}$
   f) $\sqrt[3]{64}$

3. Evaluate.
   a) $\sqrt{289}$
   b) $\sqrt{1444}$
   c) $\sqrt{3025}$
   d) $\sqrt[3]{1728}$
   e) $\sqrt[3]{5832}$
   f) $\sqrt[3]{8000}$

4. The area of a square city block is 62 500 m^2. Calculate the length of a side.

5. Taylor needs to add a lace edge to a square tablecloth. The area of the cloth is 9 m^2. What length of edging does she need?

6. The surface area of a sphere is given by the formula $SA = 4\pi r^2$. If the surface area of a beach ball is 3600π cm^2, what is the radius of the ball?

7. A cubic aquarium for five sea lions has a volume of 216 m^3. Calculate the dimensions of the aquarium.

8. The volume of a cube is 125 cm^3. Calculate the total length of all the edges.

KEY

1. a) perfect square **b)** neither **c)** perfect cube
d) perfect square **e)** both **f)** perfect cube
2. a) 16 **b)** 15 **c)** 10 **d)** 41 **e)** 8 **f)** 4
3. a) 17 **b)** 38 **c)** 55 **d)** 12 **e)** 18 **f)** 20
4. 250 m **5.** 12 m **6.** 30 cm **7.** 6 m **8.** 60 cm

2.2

~~3.2~~ Extra Practice

1. Write each expression with positive exponents.

a) c^{-4}

b) mn^{-2}

c) $3x^{-3}$

d) $4m^3n^{-2}$

e) $-2x^{-4}$

f) $-5x^{-3}y^{-2}$

2. Simplify each expression. State the answer using positive exponents.

a) $(2^{-2})(2^3)$

b) $(3^0)(3^{-3})$

c) $\dfrac{5^3}{5^{-4}}$

d) $\dfrac{(3^{-7})(4)}{(3^9)(4^3)}$

e) $(2^4)^3$

f) $(3^2)^{-4}$

g) $[(4)(2^{-3})]^{-2}$

h) $\left(\dfrac{6^2}{5^{-3}}\right)^{-3}$

3. Simplify each expression. State the answer using positive exponents.

a) $\left(2xy^2\right)\left(3x^{-1}y^0\right)$

b) $\left(-3m^2n\right)\left(-4m^4n^{-2}\right)$

c) $\dfrac{m^3n^{-2}}{\left(mn^4\right)\left(m^5n^2\right)}$

d) $(-3xy^4)^2$

e) $(4xy^{-3})^{-2}$

f) $-4x(5x)^3$

g) $\left(\dfrac{6mn^3}{4m^2n}\right)^2$

h) $\left(\dfrac{3x}{-2y^2}\right)^{-2}$

4. Simplify, then evaluate. Give the result as a fraction where necessary.

a) 5^{-2}

b) 7^0

c) $\left(\dfrac{6}{7}\right)^{-2}$

d) $-(-3)^2$

e) $\dfrac{1}{(-3)^{-2}}$

f) $3^{-1}+4^{-1}$

g) $-5(m^0+n^0)^2$

h) $\dfrac{5^{-1}+5^{-2}}{5^{-3}}$

i) $\left[\left(\dfrac{3}{4}\right)^{-2}\right]^3$

5. A bacterial culture in a lab has 500 cells. The number of cells doubles every hour. This relationship can be modelled by the equation $N = 500(2)^h$, where N is the estimated number of bacteria cells and h is the time in hours.

a) If the conditions remain ideal, how many cells will there be after 6 h?

b) How many cells were there 2 h ago?

6. Dana evaluated the expression $\left(\dfrac{1}{2}\right)^{-3} = 8$.

Is she correct? Justify your answer.

KEY

1. a) $\frac{1}{c^4}$ **b)** $\frac{m}{n^2}$ **c)** $\frac{3}{x^3}$ **d)** $\frac{4m^3}{n^2}$ **e)** $\frac{-2}{x^4}$ **f)** $\frac{-5}{x^3y^2}$

2. a) 2 **b)** $\frac{1}{3^3}$ **c)** 5^7 **d)** $\frac{1}{(3^{16})(4^2)}$ **e)** 2^{12} **f)** $\frac{1}{3^8}$

g) $\frac{2^6}{4^2}$ **h)** $\frac{1}{(6^6)(5^9)}$

3. a) $6y^2$ **b)** $\frac{12m^6}{n}$ **c)** $\frac{1}{m^3n^8}$ **d)** $9x^2y^8$

e) $\frac{y^6}{16x^2}$ **f)** $-500x^4$ **g)** $\frac{9n^4}{4m^2}$ **h)** $\frac{4y^4}{9x^2}$

4. a) $\frac{1}{25}$ **b)** 1 **c)** $\frac{49}{36}$ **d)** -9 **e)** 9 **f)** $\frac{7}{12}$

g) -20 **h)** 30 **i)** $\frac{4096}{729}$

5. a) 32 000 **b)** 125 **6.** Yes. $\left(\frac{2}{1}\right)^3 = 8$

2.3

3.3 Extra Practice

1. Use the exponent laws to simplify each expression.

a) $\left(x^{\frac{1}{2}}\right)\left(x^{\frac{7}{2}}\right)$

b) $\left(3m^{4}\right)\left(m^{\frac{1}{4}}\right)$

c) $\left[\left(x^{1.5}\right)\left(x^{2.5}\right)\right]^{0.5}$

d) $\left(\frac{5x^{3}}{20x}\right)^{\frac{1}{2}}$

e) $\left(x^{\frac{2}{3}}y^{\frac{4}{3}}\right)^{3}$

2. Simplify each expression. State the answer using positive exponents.

a) $\left(y^{-2}\right)\left(y^{\frac{5}{2}}\right)$

b) $\left(-8x^{-6}\right)^{\frac{1}{3}}$

c) $\frac{\left(x^{3}\right)^{\frac{1}{2}}}{\left(x^{\frac{5}{2}}\right)^{\frac{1}{5}}}$

d) $\left(\frac{x^{\frac{1}{4}}}{16x^{\frac{3}{4}}}\right)^{\frac{1}{2}}$

e) $\left(x^{\frac{1}{3}}y^{\frac{4}{5}}\right)^{0}\left(x^{\frac{1}{3}}\right)^{6}$

3. Evaluate without using a calculator. Leave each answer as a rational number.

a) $\frac{5^{-2}}{125^{\frac{1}{3}}}$

b) $\frac{9^{\frac{3}{2}}}{27^{2}}$

c) $\left(8^{\frac{2}{3}}\right)\left(16^{\frac{3}{2}}\right)$

d) $\left(3^{-2}\right)^{\frac{-5}{2}}$

e) $\left(125^{\frac{-1}{3}}\right)^{2}$

4. Evaluate using a calculator. Give the result to four decimal places, if necessary.

a) $\left(7^{1.2}\right)^{-3}$

b) $\left(4^{3}\right)\left(4^{\frac{3}{2}}\right)$

c) $\left(7^{3}\right)^{\frac{2}{3}}$

d) $\left(\frac{6^{2}}{3^{3}}\right)^{\frac{1}{3}}$

e) $\left[\frac{3^{2}}{(-3)^{4}}\right]^{\frac{1}{2}}$

5. The growth of 5000 bacterium cells in a lab can be modelled using the expression $N = 5000(1.5)^{\frac{h}{40}}$, where N is the number of bacteria after h hours.

a) What does the value 1.5 in the expression tell you?

b) How many bacteria are there after 40 h?

c) How many more bacteria are there after 3 h?

d) What does $h = 0$ indicate?

KEY

1. a) x^4 **b)** $3m^{\frac{17}{4}}$ **c)** x^2 **d)** $\frac{x}{2}$ **e)** x^2y^4

2. a) $y^{\frac{1}{2}}$ **b)** $\frac{-2}{x^{\frac{1}{2}}}$ **c)** x **d)** $\frac{1}{4x^{\frac{1}{4}}}$ **e)** x^2

3. a) $5^{-3} = \frac{1}{125}$ **b)** $3^{-3} = \frac{1}{27}$ **c)** $2^8 = 256$ **d)** $3^5 = 243$

e) $\left(\frac{1}{5}\right)^2 = \frac{1}{25}$

4. a) $7^{-3.6} = 0.0009$ **b)** $4^{\frac{9}{2}} = 512$ **c)** $7^2 = 49$

d) $\frac{6^{\frac{2}{3}}}{3} = 1.1006$ **e)** $3^{-1} = 0.3333$

5. a) The number of bacteria increases by 1.5 times every 40 h.
b) 7500. There are 7500 bacteria after 40 h.
c) 5154.385; 5154.385 – 5000 = 154.385. There are approximately 154 more bacteria after 3 h.
d) Example: The value $h = 0$ indicates the starting population of 5000 bacteria.

2.4

2.4 Extra Practice

1. Express each power as an equivalent radical.

a) $5^{\frac{2}{3}}$ b) $8^{0.75}$

c) $6^{\frac{3}{5}}$ d) $81^{0.5}$

★e) $\frac{1}{9^{\frac{5}{3}}}$ f) $(x^3)^{\frac{1}{4}}$

g) $\left(a^{\frac{1}{3}}\right)^2$ h) $\left[\frac{\left(x^{\frac{1}{3}}\right)}{\left(y^{\frac{1}{3}}\right)}\right]^2$

2. Express each radical as a power.

a) $\sqrt[4]{3^3}$ b) $\sqrt[3]{(5t)^4}$

c) $\sqrt[3]{x^2}$ d) $\sqrt[5]{\frac{a^2}{b^3}}$

e) $\sqrt[3]{y^{\frac{5}{2}}}$ f) $\sqrt[4]{2^3}$

3. Evaluate each expression. State the result to four decimal places, if necessary.

a) $\sqrt{0.25}$ b) $(64)^{\frac{1}{3}}$

c) $3\sqrt{12}$ d) $\sqrt{\left(\frac{5}{4}\right)^2}$

e) $4(1.2)^{\frac{3}{4}}$ f) $\frac{\sqrt[3]{16}}{\sqrt{12}}$

4. Express each mixed radical as an equivalent entire radical.

★a) $4\sqrt{5}$ b) $3\sqrt{4}$

c) $5\sqrt{13}$ d) $6.2\sqrt{10}$

e) $3.3\sqrt{16}$ f) $\frac{1}{5}\sqrt{10}$

5. Express each mixed radical as an equivalent entire radical.

a) $3\sqrt[3]{5}$ b) $7\sqrt[3]{3}$

c) $5\sqrt[3]{6}$ d) $2\sqrt[4]{7}$

e) $\frac{1}{2}\sqrt[3]{5}$ f) $1.5\sqrt[4]{10}$

6. Express each entire radical as an equivalent mixed radical.

a) $\sqrt{32}$ b) $\sqrt{44}$

c) $\sqrt{90}$ d) $\sqrt{80}$

e) $\sqrt{360}$ f) $\sqrt{475}$

7. Express each entire radical as an equivalent mixed radical.

a) $\sqrt[3]{48}$ b) $\sqrt[3]{120}$

c) $\sqrt[3]{324}$ d) $\sqrt[4]{48}$

e) $\sqrt[4]{405}$ f) $\sqrt[4]{208}$

8. Order each set of numbers from greatest to least. Then, identify the irrational numbers.

a) $0.5\sqrt{2}$ $0.\overline{7}$ $\frac{3}{4}$ $\sqrt{0.49}$

b) $\frac{2}{3}\sqrt[3]{0.343}$ $\sqrt{0.38}$ 0.62

9. Plot each set of numbers on a number line. Which of the numbers in each set is irrational?

a) $\sqrt[3]{435}$ $8.\overline{5}$ $4\sqrt{5}$ $\sqrt{64}$

b) $\frac{2\sqrt{85}}{3}$ $\sqrt[3]{216}$ $6\frac{9}{11}$ $3\sqrt{7}$

13. In the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$, r represents the radius of a sphere, in centimetres, and V is the volume of the sphere, in cubic centimetres. What is the length of the radius of a sphere with each of the following volumes? Express the answers to two decimal places.

a) 132 cm^3 b) 1896 cm^3

14. A pendulum has a length of 6 ft. The formula $T = \sqrt{\frac{4\pi^2 l}{32\ ft/s^2}}$ represents the period of the pendulum. In this formula, T is the period of the pendulum, in seconds, and l is the length of the pendulum, in feet. Calculate the period of the pendulum. Express the answer to two decimal places.

19. Without using a calculator, solve each of the following:

a) $\sqrt{\sqrt{16}}$

b) $\sqrt{\sqrt[3]{15\,625}}$

★c) $\sqrt{4 + \sqrt{19 + \sqrt{36}}}$

★d) $\sqrt[4]{13 + \sqrt[3]{22 + \sqrt[3]{125}}}$

20. Express as a power with a single rational exponent.

★a) $\sqrt[3]{\sqrt{7}}$ ★b) $\sqrt[4]{\sqrt[3]{5^2}}$

c) $\sqrt[5]{\sqrt{\frac{1}{8}}}$ d) $\sqrt[4]{\sqrt[3]{\left(\frac{2}{5}\right)^6}}$

KEY

1. a) $(\sqrt[3]{5})^2$ b) $(\sqrt[4]{8})^3$
   c) $(\sqrt[5]{6})^3$ d) $\sqrt{81}$
   e) $\frac{1}{9^{\frac{5}{3}}} = \left[\left(\frac{1}{9}\right)^{\frac{1}{3}}\right]^5 = \left(\sqrt[3]{\frac{1}{9}}\right)^5$
   f) $\sqrt[4]{x^3}$ g) $(\sqrt[3]{a})^2$
   h) $\left(\sqrt[3]{\frac{x}{y}}\right)^2$

2. a) $3^{\frac{3}{4}}$ b) $(5t)^{\frac{4}{3}}$
   c) $x^{\frac{2}{3}}$ d) $\left(\frac{a^2}{b^3}\right)^{\frac{1}{5}}$ or $\frac{a^{\frac{2}{5}}}{b^{\frac{3}{5}}}$
   e) $y^{\frac{5}{6}}$ f) $2^{\frac{3}{a}}$

3. a) 0.5 b) 4
   c) 10.3923 d) 1.25
   e) 4.5861 f) 0.7274

4. a) $4\sqrt{5} = \sqrt{(4^2)\sqrt{5}}$
   $= \sqrt{(16)(5)}$
   $= \sqrt{80}$
   b) $\sqrt{36}$
   c) $\sqrt{325}$ d) $\sqrt{384.4}$
   e) $\sqrt{174.24}$ f) $\sqrt{\frac{10}{25}}$ or $\sqrt{\frac{2}{5}}$

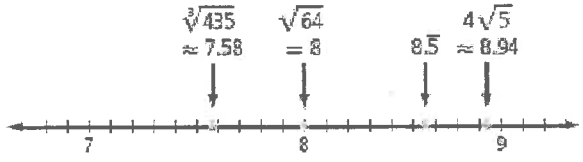
5. a) $\sqrt[3]{135}$ b) $\sqrt[3]{1029}$
   c) $\sqrt[3]{750}$ d) $\sqrt[4]{112}$
   e) $\sqrt[3]{\frac{5}{8}}$ f) $\sqrt[4]{50.625}$

6. a) $4\sqrt{2}$ b) $2\sqrt{11}$
   c) $3\sqrt{10}$ d) $4\sqrt{5}$
   e) $6\sqrt{10}$ f) $5\sqrt{19}$

7. a) $2\sqrt[3]{6}$ b) $2\sqrt[3]{15}$
   c) $3\sqrt[3]{12}$ d) $2\sqrt[4]{3}$
   e) $3\sqrt[4]{5}$ f) $2\sqrt[4]{13}$

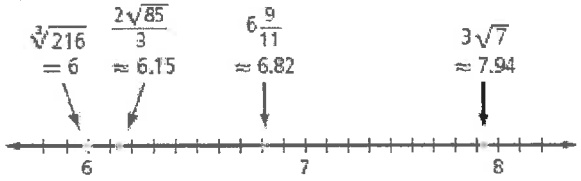
8. a) $0.\overline{7}, \frac{3}{4}, 0.5\sqrt{2}, \sqrt{0.49}$; $0.5\sqrt{2}$ is an irrationa number.
   b) $\sqrt[3]{0.343}, \frac{2}{3}, 0.62, \sqrt{0.38}$; $\sqrt{0.38}$ is an irrational number.

9. a)



$\sqrt[3]{435}$ and $4\sqrt{5}$ are irrational numbers.

b)



$\frac{2\sqrt{85}}{3}$ and $3\sqrt{7}$ are irrational numbers.

13. a) approximately 3.16 cm
    b) approximately 7.68 cm

14. 2.72 s

19. a) 2 b) 5
    c) $\sqrt{4 + \sqrt{19 + \sqrt{36}}} = \sqrt{4 + \sqrt{19 + 6}}$
    $= \sqrt{4 + \sqrt{25}}$
    $= \sqrt{4 + 5}$
    $= \sqrt{9}$
    $= 3$
    d) $\sqrt[4]{13 + \sqrt[3]{22 + \sqrt[3]{125}}} = \sqrt[4]{13 + \sqrt[3]{22 + 5}}$
    $= \sqrt[4]{13 + \sqrt[3]{27}}$
    $= \sqrt[4]{13 + 3}$
    $= \sqrt[4]{16}$
    $= 2$

20. a) $\sqrt[3]{\sqrt{7}} = (\sqrt{7})^{\frac{1}{3}}$
    $= \left(7^{\frac{1}{2}}\right)^{\frac{1}{3}}$
    $= 7^{\frac{1}{6}}$
    b) $\sqrt[4]{\sqrt[3]{5^2}} = (\sqrt[3]{5^2})^{\frac{1}{4}}$
    $= \left[(5^2)^{\frac{1}{3}}\right]^{\frac{1}{4}}$
    $= 5^{(2)(\frac{1}{3})(\frac{1}{4})}$
    $= 5^{\frac{1}{6}}$
    c) $\left(\frac{1}{8}\right)^{\frac{1}{10}}$ d) $\left(\frac{2}{5}\right)^{\frac{1}{2}}$