

FMP10

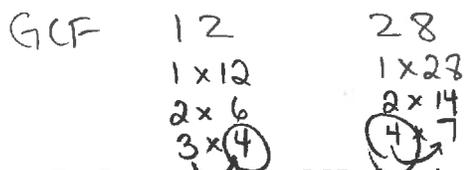
Name: _____

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Unit 5B: Factoring
5B.1 Common Factors

Greatest Common Factor (GCF):

Largest factor shared by 2 or more terms.



GCF 4

Ex. Determine the GCF of each pair of terms:

a) $16x^2y$ and $24x^2y^3$

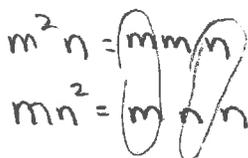
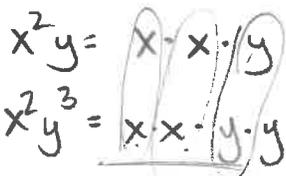
GCF = $8x^2y$

b) $5m^2n$ and $15mn^2$

GCF = $5mn$

c) $48ab^3c$ and $36a^2b^2c^2d$

GCF = $12ab^2c$



Recall distribution: $2x(3x^2 - 4) = 2x(3x^2) - 2x(4)$ ②

Factoring is the reverse process: $= 6x^3 - 8x$ ①

To factor means to write as a product.

Ex. Write each polynomial in factored form.

GCF = 4r

a) $\frac{16r^2}{4r} - \frac{20r}{4r}$

$4r(4r - 5)$

Steps

- ① Identify GCF
 - ② Divide GCF out
 - ③ write as a product of factor.
- (GCF and 'leftovers')

c) $\frac{27r^2s^2}{9rs^2} - \frac{18r^3s^2}{9rs^2} - \frac{36rs^3}{9rs^2}$

$9rs^2(3r - 2r^2 - 4s)$

b) $\frac{4x^3y^2}{2x^2} - \frac{14x^2}{2x^2}$

$= 2x^2(2xy^2 - 7)$

d) $12n^5p^4 + 8n^4p^3 - 4n^3p^2$

$4n^3p^2(3n^2p^2 + 2np - 1)$

Recall double distribution: $(x-3)(2x+y) = x(2x+y) - 3(2x+y)$ Step 1
 $= 2x^2 + xy - 6x - 3y$ Step 2

We can reverse step 1 by factoring a binomial factor:

Ex. Factor each polynomial.

a) $\frac{3x(x-4)}{(x-4)} + \frac{5(x-4)}{(x-4)}$
 $= (x-4)(3x+5)$

b) $\frac{4(x+5)}{x+5} - \frac{3x(x+5)}{x+5}$
 $(x+5)(4-3x)$

We can reverse step 2 by group factoring:

Ex. Factor each polynomial.

a) $\frac{y^2 + 8xy}{y} + \frac{2y + 16x}{2}$
 $y(y+8x) + 2(y+8x)$
 $(y+8x)(y+2)$

b) $\frac{a^2 - a}{a-1} + \frac{3ab - 3b}{a-1}$
 $a(a-1) + 3b(a-1)$
 $(a+3b)(a-1)$

Ex. Paula has 18 toonies, 30 loonies, and 48 quarters. She wants to group her money so that each group has the same number of each coin and there are no coins leftover.

a) What is the maximum number of groups she can make?

GCF = 6 6 group.

b) How many of each coin will be in each group?

$\frac{18}{6} \quad \frac{30}{6} \quad \frac{48}{6}$
 $3t + 5l + 8q$

c) How much money will each group be worth?

$3(2) + 5(1) + 8(0.25)$
 $= 6 + 5 + 2$
 $= \$13$

5B.3 Decomposition Factoring

Recall:

$$\begin{aligned} (2x-3)(x+2) &= 2x(x+2) - 3(x+2) \\ &= 2x^2 + 4x - 3x - 6 \\ &= 2x^2 + x - 6 \end{aligned}$$

To start factoring we must split the middle term

Decomposition Factoring

Factor. $3x^2 + 8x + 4$

	x AC		+B
	x 12		+ 8
$3x^2 + 2x + 6x + 4$	1	12	13
$x(3x+2) + 2(3x+2)$	2	6	8
$(3x+2)(x+2)$	3	4	7

Step 1: Check for a GCF

Step 2: Find two integers with

- A product of $3 \times 4 = 12$
- A sum of 8

Step 3: Split (decompose) the middle term into two parts using the integers from step 1

Step 4: Factor by grouping

Check?

$$Ax^2 + Bx + C = 0$$

Ex. Factor each trinomial, if possible:

a) $2x^2 + 7x - 4$

$$\begin{aligned} &2x^2 - x + 8x - 4 \\ &x(2x-1) + 4(2x-1) \\ &(2x-1)(x+4) \end{aligned}$$

	x AC		+B
	x -8		+ 7
	-1	8	7
	1	-8	-7
	-2	4	2
	2	-4	-2

c) $24x^2 - 30x - 9 = 3(8x^2 - 10x - 3)$

	x AC		+B
	-24		+ -10
-1	24	23	
1	-24	-23	
-2	12	10	
2	-12	-10	

$$= 3(8x^2 + 2x - 12x - 3)$$

$$= 3(2x(4x+1) - 3(4x+1))$$

$$= 3(4x+1)(2x-3)$$

b) $3x^2 + 2x + 4$

Prime

Cannot be factored

	x AC		+B
	x 12		+ 2
1	12	13	
2	6	8	
3	4	7	

d) $6x^2 - 5xy + y^2 = 6x^2 - 2xy - 3xy + y^2$

	x AC		+B
	x 6		+ (-5)
-2	-3	-5	

$$2x(3x-y) - y(3x-y)$$

$$(3x-y)(2x-y)$$

e) $-3a^3b - 51a^2b - 30ab$
 $-3ab(a^2 + 17a + 10)$
 no more factoring possible

x	10	+ 17
	1 10	11
	2 5	7

f) $2y^2 + 7xy + 3x^2$

Ex. Identify all values of q that allows each trinomial to be factored:

a) $2x^2 + qx + 5$

x	10	+ q
	1 10	11
	2 5	7
	-1 -10	-11
	-2 -5	-7

b) $3x^2 + qxy - 4y^2$

x	-12	+ q
	-1 12	11
	1 -12	-11
	-2 6	4
	2 -6	-4
	-3 4	1
	3 -4	-1

Ex. Identify two values of q that allows each trinomial to be factored:

a) $2a^2 - 3a + q$

x	2q	+ -3
	2	-2 -1
	-4	-4 -1
	-28	4 -7

$2q = 2$
 $q = 1$
 $2q = -4$
 $q = -2$
 $2q = -28$
 $q = -14$

b) $5a^2 + 2ab + qb^2$

x	5q	+ 2
	-80	10 -8
	-8	4 -2
	-15	5 -3

$5q = -80$
 $q = -16$
 ~~$5q = 8$
 $q = 8/5$~~
 ~~$5q = -8$
 $q = -8/5$~~
 $5q = -15$
 $q = -3$

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5B.3 Factoring Special Trinomials

Ex. Factor:

$$\begin{array}{r|l} x-36 & 0 \\ -66 & 0 \end{array}$$

$$\begin{aligned} 4x^2 - 9 &= 4x^2 + 0x - 9 \\ &= 4x^2 + 6x - 6x - 9 \\ &= 2x(2x+3) - 3(2x+3) \\ &= (2x+3)(2x-3) \end{aligned}$$

$$\begin{array}{r|l} x-25 & +0 \\ -55 & \end{array}$$

$$\begin{aligned} a^2 - 25b^2 &= a^2 + 0ab - 25b^2 \\ &= a^2 - 5ab + 5ab - 25b^2 \\ &= a(a-5b) + 5b(a-5b) \\ &= (a-5b)(a+5b) \end{aligned}$$

Difference of Squares:

A binomial expression involving subtraction of two squares.

Pattern can be applied to both multiplication of conjugates and factoring of difference of squares.

Ex. Multiply:

Diff of Squares

$$\begin{aligned} (3x-4)(3x+4) \\ (3x)^2 - (4)^2 \\ 9x^2 - 16 \end{aligned}$$

$$\begin{aligned} (a^2+7b)(a^2-7b) \\ (a^2)^2 - (7b)^2 \\ a^4 - 49b^2 \end{aligned}$$

$$\begin{aligned} 2(6q-1)(6q+1) \\ = 2(36q^2 - 1) \\ = 72q^2 - 2 \end{aligned}$$

Ex. Factor:

$$\sqrt{x^2-81}$$

$$(x-9)(x+9)$$

$$\begin{aligned} -16c^2 + 25a^2 \\ 25a^2 - 16c^2 \\ (5a+4c)(5a-4c) \end{aligned}$$

$m^2 + 16$
Prime

$$\begin{aligned} 7g^3h^2 - 28g^5 \\ 7g^3(h^2 - 4g^2) \\ 7g^3(h+2g)(h-2g) \end{aligned}$$

Ex. Factor:

$$\begin{array}{r|l} x & + \\ 36 & 12 \\ 66 & 12 \end{array}$$

$$\begin{aligned} 4x^2 + 12x + 9 \\ = \frac{4x^2}{2x} + \frac{6x}{2x} + \frac{6x}{3} + \frac{9}{3} \\ = 2x(2x+3) + 3(2x+3) \\ (2x+3)(2x+3) \\ (2x+3)^2 \end{aligned}$$

$$\begin{array}{r|l} x & + \\ 16 & -8 \\ -4 & -4 \end{array}$$

$$\begin{aligned} x^2 - 8x + 16 \\ \frac{x^2-4x}{x} - \frac{4x+16}{-4} \\ x(x-4) - 4(x-4) \\ (x-4)^2 \end{aligned}$$

Perfect Square Trinomial: The results of binomial squares.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Ex. Multiply:

$$(x+5)^2$$

$$x^2 + 2x5 + (5)^2$$

$$x^2 + 10x + 25$$

$$(3q)^2 - 2(3q)2 + 2^2$$

$$9q^2 - 12q + 4$$

$$3(2a+c)^2$$

$$3(2a)^2 + 2(2a)(c) + c^2$$

$$3(4a^2 + 4ac + c^2)$$

$$12a^2 + 12ac + 3c^2$$

How can we identify a perfect square trinomial?

The middle term should equal double the product of the square roots of the end terms

→ end terms are perfect squares.

Ex. Determine the values of q that would make each trinomial a perfect square:

$$\sqrt{x^2} + qx + \sqrt{1}$$

$$2x(1)$$

$$\frac{2x}{\uparrow}$$

$$q = \pm 2$$

$$\sqrt{9x^2} + qx + \sqrt{16}$$

$$2(3x)4$$

$$24x$$

$$q = \pm 24$$

$$49a^2 + qab + 36b^2$$

$$2(7a)(6b)$$

$$84ab$$

$$q = \pm 84$$

Ex. Determine if each trinomial is a perfect square, if yes factor using the pattern:

$$x^2 + 6x + 9$$

$$\begin{array}{l} 2(x)(3) \\ \downarrow \\ 6x \end{array} \checkmark$$

$$(x+3)^2$$

$$c^2 - 14c + 36$$

$$\begin{array}{l} 2(c)(6) \\ \downarrow \\ 12c \end{array} \times$$

$$25p^2 - 40pq + 16q^2$$

$$\begin{array}{l} 2(5p)(4q) \\ \downarrow \\ 40pq \end{array}$$

$$(5p-4q)^2$$

$$4x^2 + 10x + 25$$

$$\begin{array}{l} 2(2x)(5) \\ \downarrow \\ 20x \end{array} \times$$